Chapter 4

ALGEBRAIC FRACTIONS

EXERCISE 4A

1 **a** $\frac{c}{2}$
$= \frac{6}{2}$
$= 3$

b $\frac{c}{a}$
$= \frac{6}{3}$
$= 2$

c $\frac{a}{c}$
$= \frac{3}{6}$
$= \frac{1}{2}$

d $\frac{c}{b-a}$
$= \frac{6}{2-3}$
$= \frac{6}{-1}$
$= -6$

e $\frac{a+c}{b}$
$= \frac{3+6}{2}$
$= \frac{9}{2}$

f $\frac{ab}{c}$
$= \frac{3 \times 2}{6}$
$= \frac{6}{6}$
$= 1$

g $\frac{a^2}{b}$
$= \frac{3^2}{2}$
$= \frac{9}{2}$

h $\frac{c^2}{a}$
$= \frac{6^2}{3}$
$= \frac{36}{3}$
$= 12$

i $\frac{ab^2}{c}$
$= \frac{3 \times 2^2}{6}$
$= \frac{3 \times 4}{6}$
$= \frac{12}{6}$
$= 2$

j $\frac{(ab)^2}{c}$
$= \frac{(3 \times 2)^2}{6}$
$= \frac{6^2}{6}$
$= \frac{36}{6}$
$= 6$

2 **a** $\frac{c}{a}$
$= \frac{-4}{2}$
$= -2$

b $\frac{a}{c}$
$= \frac{2}{-4}$
$= -\frac{1}{2}$

c $\frac{-1}{b}$
$= \frac{-1}{-3}$
$= \frac{1}{3}$

d $\frac{c^2}{a}$
$= \frac{(-4)^2}{2}$
$= \frac{16}{2}$
$= 8$

e $\frac{c}{a+b}$
$= \frac{-4}{2+(-3)}$
$= \frac{-4}{2-3}$
$= \frac{-4}{-1}$
$= 4$

f $\frac{a-c}{2b}$
$= \frac{2-(-4)}{2 \times (-3)}$
$= \frac{2+4}{-6}$
$= \frac{6}{-6}$
$= -1$

g $\frac{b}{c-a}$
$= \frac{-3}{-4-2}$
$= \frac{-3}{-6}$
$= \frac{1}{2}$

h $\frac{a-c}{a+c}$
$= \frac{2-(-4)}{2+(-4)}$
$= \frac{2+4}{2-4}$
$= \frac{6}{-2}$
$= -3$

i $\frac{c-a}{b^2}$
$= \frac{-4-2}{(-3)^2}$
$= \frac{-6}{9}$
$= -\frac{2}{3}$

j $\frac{a^2}{c-b}$
$= \frac{2^2}{-4-(-3)}$
$= \frac{4}{-4+3}$
$= \frac{4}{-1}$
$= -4$

3 **a** $\frac{2z}{y}$
$= \frac{2 \times 5}{-2}$
$= -5$

b $\frac{3x}{z+1}$
$= \frac{3 \times 3}{5+1}$
$= \frac{9}{6}$
$= \frac{3}{2}$

c $\frac{y+z}{x}$
$= \frac{-2+5}{3}$
$= \frac{3}{3}$
$= 1$

d $\frac{4z+y}{x^2}$
$= \frac{4 \times 5 - 2}{3^2}$
$= \frac{18}{9}$
$= 2$

e $\frac{\sqrt{z^2-x^2}}{y}$
$= \frac{\sqrt{5^2-3^2}}{-2}$
$= \frac{\sqrt{25-9}}{-2}$
$= \frac{\sqrt{16}}{-2}$
$= \frac{4}{-2}$
$= -2$

EXERCISE 4B.1

1 **a** $\dfrac{2a}{4} = \dfrac{{}^{1}\cancel{2} \times a}{{}_{2}\cancel{4}} = \dfrac{a}{2}$

b $\dfrac{4m}{2} = \dfrac{{}^{2}\cancel{4} \times m}{{}_{1}\cancel{2}} = \dfrac{2 \times m}{1} = 2m$

c $\dfrac{6a}{a} = \dfrac{6 \times \cancel{a}^{1}}{{}_{1}\cancel{a}} = \dfrac{6}{1} = 6$

d $\dfrac{6a}{2a} = \dfrac{{}^{3}\cancel{6} \times \cancel{a}^{1}}{{}_{1}\cancel{2} \times \cancel{a}_{1}} = \dfrac{3}{1} = 3$

e $\dfrac{2a^2}{a} = \dfrac{2 \times a \times \cancel{a}^{1}}{{}_{1}\cancel{a}} = \dfrac{2 \times a}{1} = 2a$

f $\dfrac{2x^3}{2x} = \dfrac{{}^{1}\cancel{2} \times x \times x \times \cancel{x}^{1}}{{}_{1}\cancel{2} \times \cancel{x}_{1}} = \dfrac{x \times x}{1} = x^2$

g $\dfrac{2x^3}{x^2} = \dfrac{2 \times x \times {}^{1}\cancel{x} \times \cancel{x}^{1}}{{}_{1}\cancel{x} \times \cancel{x}_{1}} = \dfrac{2x}{1} = 2x$

h $\dfrac{2x^3}{x^3} = \dfrac{2 \times \cancel{x}^{1} \times \cancel{x}^{1} \times \cancel{x}^{1}}{{}_{1}\cancel{x} \times \cancel{x}_{1} \times \cancel{x}_{1}} = \dfrac{2}{1} = 2$

i $\dfrac{2a^2}{4a^3} = \dfrac{{}^{1}\cancel{2} \times \cancel{a}^{1} \times \cancel{a}^{1}}{{}_{2}\cancel{4} \times a \times \cancel{a}_{1} \times \cancel{a}_{1}} = \dfrac{1}{2 \times a} = \dfrac{1}{2a}$

j $\dfrac{8m^2}{4m} = \dfrac{{}^{2}\cancel{8} \times m \times \cancel{m}^{1}}{{}_{1}\cancel{4} \times \cancel{m}_{1}} = \dfrac{2 \times m}{1} = 2m$

k $\dfrac{4a^2}{a^2} = \dfrac{4 \times \cancel{a}^{1} \times \cancel{a}^{1}}{{}_{1}\cancel{a} \times \cancel{a}_{1}} = \dfrac{4}{1} = 4$

l $\dfrac{6t}{3t^2} = \dfrac{{}^{2}\cancel{6} \times \cancel{t}^{1}}{{}_{1}\cancel{3} \times \cancel{t}_{1} \times t} = \dfrac{2}{t}$

m $\dfrac{4d^2}{2d} = \dfrac{{}^{2}\cancel{4} \times d \times \cancel{d}^{1}}{{}_{1}\cancel{2} \times \cancel{d}_{1}} = \dfrac{2 \times d}{1} = 2d$

n $\dfrac{ab^2}{2ab} = \dfrac{{}^{1}\cancel{a} \times b \times \cancel{b}^{1}}{2 \times \cancel{a}_{1} \times \cancel{b}_{1}} = \dfrac{b}{2}$

o $\dfrac{4ab^2}{6a^2b} = \dfrac{{}^{2}\cancel{4} \times {}^{1}\cancel{a} \times b \times \cancel{b}^{1}}{{}_{3}\cancel{6} \times a \times \cancel{a}_{1} \times \cancel{b}_{1}} = \dfrac{2 \times b}{3 \times a} = \dfrac{2b}{3a}$

2 **a** $\dfrac{2t}{2} = \dfrac{{}^{1}\cancel{2} \times t}{\cancel{2}_{1}} = \dfrac{t}{1} = t$

b $\dfrac{2+t}{2}$ cannot be simplified as $2+t$ is a sum, not a product.

c $\dfrac{xy}{x} = \dfrac{{}^{1}\cancel{x} \times y}{{}_{1}\cancel{x}} = \dfrac{y}{1} = y$

d $\dfrac{x+y}{x}$ cannot be simplified as $x+y$ is a sum, not a product.

e $\dfrac{ac}{bc} = \dfrac{a \times \cancel{c}^{1}}{b \times \cancel{c}_{1}} = \dfrac{a}{b}$

f $\dfrac{a+c}{b+c}$ cannot be simplified as $a+c$ and $b+c$ are sums, not products.

g $\dfrac{2a^2}{4a} = \dfrac{{}^{1}\cancel{2} \times a \times \cancel{a}^{1}}{{}_{2}\cancel{4} \times \cancel{a}_{1}} = \dfrac{a}{2}$

h $\dfrac{5a}{9b}$ cannot be simplified as there are no common factors in the numerator and denominator.

i $\dfrac{14c}{8d} = \dfrac{{}^{7}\cancel{14} \times c}{{}_{4}\cancel{8} \times d} = \dfrac{7c}{4d}$

3 **a** $\dfrac{(2a)^2}{a^2} = \dfrac{2a \times 2a}{a \times a} = \dfrac{4 \times \cancel{a}^{1} \times \cancel{a}^{1}}{{}_{1}\cancel{a} \times \cancel{a}_{1}} = 4$

b $\dfrac{b}{(3b)^2} = \dfrac{b}{3b \times 3b} = \dfrac{\cancel{b}^{1}}{3 \times \cancel{b}_{1} \times 3 \times b} = \dfrac{1}{9b}$

c $\dfrac{2a}{(4a)^2} = \dfrac{2a}{4a \times 4a} = \dfrac{{}^{1}\cancel{2} \times \cancel{a}^{1}}{{}_{2}\cancel{4} \times a \times 4 \times \cancel{a}_{1}} = \dfrac{1}{8a}$

d $\dfrac{(4n)^2}{8n} = \dfrac{4n \times 4n}{8 \times n} = \dfrac{{}^{2}\cancel{16} \times n \times \cancel{n}^{1}}{{}_{1}\cancel{8} \times \cancel{n}_{1}} = 2n$

e $\dfrac{(-a)^2}{a} = \dfrac{-a \times -a}{a} = \dfrac{a \times \cancel{a}^{1}}{\cancel{a}_{1}} = a$

f $\dfrac{a^2}{(-a)^2} = \dfrac{a \times a}{-a \times -a} = \dfrac{{}^{1}\cancel{a} \times \cancel{a}^{1}}{{}_{1}\cancel{a} \times \cancel{a}_{1}} = 1$

g $\dfrac{(-2a)^2}{4} = \dfrac{(-2a) \times (-2a)}{4} = \dfrac{{}^{1}\cancel{4} \times a \times a}{{}_{1}\cancel{4}} = a^2$

h $\dfrac{(-3n)^2}{6n} = \dfrac{(-3n) \times (-3n)}{6 \times n} = \dfrac{{}^{3}\cancel{9} \times n \times \cancel{n}^{1}}{{}_{2}\cancel{6} \times \cancel{n}_{1}} = \dfrac{3n}{2}$

4 **a** $\dfrac{2(a+3)}{2} = \dfrac{{}^{1}\cancel{2}(a+3)}{{}_{1}\cancel{2}} = a+3$

b $\dfrac{4(x+5)}{2} = \dfrac{{}^{2}\cancel{4}(x+5)}{{}_{1}\cancel{2}} = 2(x+5)$

c $\dfrac{28(a-3)}{4} = \dfrac{{}^{7}\cancel{28}(a-3)}{{}_{1}\cancel{4}} = 7(a-3)$

d $\dfrac{4(k-2)}{12} = \dfrac{{}^{1}\cancel{4}(k-2)}{\cancel{12}_{3}} = \dfrac{k-2}{3}$

e $\dfrac{2}{8(g+5)} = \dfrac{{}^{1}\cancel{2}}{{}_{4}\cancel{8}(g+5)} = \dfrac{1}{4(g+5)}$

f $\dfrac{15}{3(t-1)} = \dfrac{{}^{5}\cancel{15}}{{}_{1}\cancel{3}(t-1)} = \dfrac{5}{t-1}$

g $\dfrac{20(t+6)}{12} = \dfrac{{}^{5}\cancel{20}(t+6)}{\cancel{12}_{3}} = \dfrac{5(t+6)}{3}$

h $\dfrac{10}{25(k+4)} = \dfrac{{}^{2}\cancel{10}}{{}_{5}\cancel{25}(k+4)} = \dfrac{2}{5(k+4)}$

5 **a** $\dfrac{(x+y)^2}{x+y} = \dfrac{(x+y)\cancel{(x+y)}^{1}}{\cancel{x+y}_{1}} = x+y$

b $\dfrac{x-3}{(x-3)^2} = \dfrac{\cancel{x-3}^{1}}{{}_{1}\cancel{(x-3)}(x-3)} = \dfrac{1}{x-3}$

c $\dfrac{(a-1)^2}{5(a-1)} = \dfrac{(a-1)\cancel{(a-1)}^{1}}{5\cancel{(a-1)}_{1}} = \dfrac{a-1}{5}$

d $\dfrac{2(x+2)}{(x+2)^2} = \dfrac{2\cancel{(x+2)}^{1}}{(x+2)\cancel{(x+2)}_{1}} = \dfrac{2}{x+2}$

e $\dfrac{(a+5)^2}{3(a+5)} = \dfrac{(a+5)\cancel{(a+5)}^{1}}{3\cancel{(a+5)}_{1}} = \dfrac{a+5}{3}$

f $\dfrac{3(b-4)}{6(b-4)^2} = \dfrac{{}^{1}\cancel{3}\cancel{(b-4)}^{1}}{{}_{2}\cancel{6}(b-4)\cancel{(b-4)}_{1}} = \dfrac{1}{2(b-4)}$

g $\dfrac{8(p+q)^2}{12(p+q)} = \dfrac{{}^{2}\cancel{8}(p+q)\cancel{(p+q)}^{1}}{{}_{3}\cancel{12}\cancel{(p+q)}_{1}} = \dfrac{2(p+q)}{3}$

h $\dfrac{24(r-2)}{15(r-2)^2} = \dfrac{{}^{8}\cancel{24}\cancel{(r-2)}^{1}}{{}_{5}\cancel{15}(r-2)\cancel{(r-2)}_{1}} = \dfrac{8}{5(r-2)}$

6 **a** $\dfrac{{}^{1}\cancel{(x+4)}(x+2)}{9\cancel{(x+4)}_{1}} = \dfrac{(x+2)}{9} = \dfrac{x+2}{9}$

b $\dfrac{12\cancel{(a-3)}^{1}}{{}_{1}\cancel{(a-3)}(a+1)} = \dfrac{12}{(a+1)} = \dfrac{12}{a+1}$

c $\dfrac{(x+y)\cancel{(x-y)}^{1}}{3\cancel{(x-y)}_{1}} = \dfrac{(x+y)}{3} = \dfrac{x+y}{3}$

d $\dfrac{{}^{1}\cancel{2}\cancel{x}\overset{1}{\cancel{y}}\cancel{(x-y)}^{1}}{{}_{3}\cancel{6}\cancel{x}\underset{1}{\cancel{(x-y)}}_{1}}$
$= \dfrac{y}{3}$

e $\dfrac{{}^{1}\cancel{5}\overset{1}{\cancel{(y+2)}}(y-3)}{{}_{3}\cancel{15}\underset{1}{\cancel{(y+2)}}_{1}}$
$= \dfrac{(y-3)}{3}$
$= \dfrac{y-3}{3}$

f $\dfrac{{}^{1}x(x+1)\cancel{(x+2)}^{1}}{{}_{3}\cancel{x}\underset{1}{\cancel{(x+2)}}_{1}}$
$= \dfrac{(x+1)}{3}$
$= \dfrac{x+1}{3}$

EXERCISE 4B.2

1 a $\dfrac{2x+4}{2}$
$= \dfrac{2(x+2)}{2}$ ← HCF is 2
$= \dfrac{{}^{1}\cancel{2}(x+2)}{{}_{1}\cancel{2}}$
$= x+2$

b $\dfrac{3x-6}{3}$
$= \dfrac{3(x-2)}{3}$ ← HCF is 3
$= \dfrac{{}^{1}\cancel{3}(x-2)}{{}_{1}\cancel{3}}$
$= x-2$

c $\dfrac{3x+6}{6}$
$= \dfrac{3(x+2)}{6}$ ← HCF is 3
$= \dfrac{{}^{1}\cancel{3}(x+2)}{{}_{2}\cancel{6}}$
$= \dfrac{x+2}{2}$

d $\dfrac{4x-20}{8}$
$= \dfrac{4(x-5)}{8}$ ← HCF is 4
$= \dfrac{{}^{1}\cancel{4}(x-5)}{{}_{2}\cancel{8}}$
$= \dfrac{x-5}{2}$

e $\dfrac{4y+12}{12}$
$= \dfrac{4(y+3)}{12}$ ← HCF is 4
$= \dfrac{{}^{1}\cancel{4}(y+3)}{{}_{3}\cancel{12}}$
$= \dfrac{y+3}{3}$

f $\dfrac{6x-30}{4}$
$= \dfrac{6(x-5)}{4}$ ← HCF is 6
$= \dfrac{{}^{3}\cancel{6}(x-5)}{{}_{2}\cancel{4}}$
$= \dfrac{3(x-5)}{2}$
$= \dfrac{3x-15}{2}$

g $\dfrac{ax+bx}{x}$
$= \dfrac{x(a+b)}{x}$ ← HCF is x
$= \dfrac{{}^{1}\cancel{x}(a+b)}{{}_{1}\cancel{x}}$
$= a+b$

h $\dfrac{ax+bx}{cx+dx}$
$= \dfrac{x(a+b)}{x(c+d)}$ ← HCF is x ← HCF is x
$= \dfrac{{}^{1}\cancel{x}(a+b)}{{}_{1}\cancel{x}(c+d)}$
$= \dfrac{a+b}{c+d}$

2 a $\dfrac{4x+6}{6}$
$= \dfrac{2(2x+3)}{6}$ ← HCF is 2
$= \dfrac{{}^{1}\cancel{2}(2x+3)}{{}_{3}\cancel{6}}$
$= \dfrac{2x+3}{3}$

b $\dfrac{4x+6}{5}$
$= \dfrac{2(2x+3)}{5}$ ← HCF is 2
which cannot be simplified

c $\dfrac{6a-3}{2}$
$= \dfrac{3(2a-1)}{2}$ ← HCF is 3
which cannot be simplified

d $\dfrac{6a-3}{3}$
$= \dfrac{3(2a-1)}{3}$ ← HCF is 3
$= \dfrac{{}^{1}\cancel{3}(2a-1)}{{}_{1}\cancel{3}}$
$= 2a-1$

e $\dfrac{6a+2}{4}$
$= \dfrac{2(3a+1)}{4}$ ← HCF is 2
$= \dfrac{{}^{1}\cancel{2}(3a+1)}{{}_{2}\cancel{4}}$
$= \dfrac{3a+1}{2}$

f $\dfrac{3b+9}{2}$
$= \dfrac{3(b+3)}{2}$ ← HCF is 3
which cannot be simplified

g
$$\frac{3b+9}{6}$$
$$= \frac{3(b+3)}{6} \longleftarrow \text{HCF is } 3$$
$$= \frac{{}^{1}\cancel{3}(b+3)}{{}_{2}\cancel{6}}$$
$$= \frac{b+3}{2}$$

h
$$\frac{8b-12}{6}$$
$$= \frac{4(2b-3)}{6} \longleftarrow \text{HCF is } 3$$
$$= \frac{{}^{2}\cancel{4}(2b-3)}{{}_{3}\cancel{6}}$$
$$= \frac{2(2b-3)}{3}$$
$$= \frac{4b-6}{3}$$

3 **a**
$$\frac{2x+8}{x+4}$$
$$= \frac{2\cancel{(x+4)}^{1}}{\cancel{x+4}_{1}} \longleftarrow \text{HCF is } 2$$
$$= 2$$

b
$$\frac{2x-4}{3x-6}$$
$$= \frac{2\cancel{(x-2)}^{1}}{3\cancel{(x-2)}_{1}} \longleftarrow \text{HCF is } 2 \longleftarrow \text{HCF is } 3$$
$$= \frac{2}{3}$$

c
$$\frac{3x+6}{4x+8}$$
$$= \frac{3(x+2)}{4(x+2)} \longleftarrow \text{HCF is } 3 \longleftarrow \text{HCF is } 4$$
$$= \frac{3\cancel{(x+2)}^{1}}{4\cancel{(x+2)}_{1}}$$
$$= \frac{3}{4}$$

d
$$\frac{5x-15}{3x-9}$$
$$= \frac{5(x-3)}{3(x-3)} \longleftarrow \text{HCF is } 5 \longleftarrow \text{HCF is } 3$$
$$= \frac{5\cancel{(x-3)}^{1}}{3\cancel{(x-3)}_{1}}$$
$$= \frac{5}{3}$$

e
$$\frac{ax+bx}{a+b}$$
$$= \frac{x(a+b)}{a+b} \longleftarrow \text{HCF is } x$$
$$= \frac{x\cancel{(a+b)}^{1}}{\cancel{(a+b)}_{1}}$$
$$= x$$

f
$$\frac{16x-8}{20x-10}$$
$$= \frac{8(2x-1)}{10(2x-1)} \longleftarrow \text{HCF is } 8 \longleftarrow \text{HCF is } 10$$
$$= \frac{{}^{4}\cancel{8}\cancel{(2x-1)}^{1}}{{}_{5}\cancel{10}\cancel{(2x-1)}_{1}}$$
$$= \frac{4}{5}$$

g
$$\frac{a+b}{ay+by}$$
$$= \frac{a+b}{y(a+b)} \longleftarrow \text{HCF is } y$$
$$= \frac{\cancel{(a+b)}^{1}}{y\cancel{(a+b)}_{1}}$$
$$= \frac{1}{y}$$

h
$$\frac{ax+bx}{ay+by}$$
$$= \frac{x(a+b)}{y(a+b)} \longleftarrow \text{HCF is } x \longleftarrow \text{HCF is } y$$
$$= \frac{x\cancel{(a+b)}^{1}}{y\cancel{(a+b)}_{1}}$$
$$= \frac{x}{y}$$

i
$$\frac{4x^2+8x}{x+2}$$
$$= \frac{4x(x+2)}{x+2} \longleftarrow \text{HCF is } 4x$$
$$= \frac{4 \times x \times \cancel{(x+2)}^{1}}{\cancel{(x+2)}_{1}}$$
$$= 4x$$

j
$$\frac{3x^2+9x}{x+3}$$
$$= \frac{3x(x+3)}{x+3} \longleftarrow \text{HCF is } 3x$$
$$= \frac{3 \times x \times \cancel{(x+3)}^{1}}{\cancel{(x+3)}_{1}}$$
$$= 3x$$

k
$$\frac{5x^2-5xy}{7x-7y}$$
$$= \frac{5x(x-y)}{7(x-y)} \longleftarrow \text{HCF is } 5x \longleftarrow \text{HCF is } 7$$
$$= \frac{5 \times x \times \cancel{(x-y)}^{1}}{7\cancel{(x-y)}_{1}}$$
$$= \frac{5x}{7}$$

l
$$\frac{9b^2-9ab}{12b-12a}$$
$$= \frac{9b(b-a)}{12(b-a)} \longleftarrow \text{HCF is } 9b \longleftarrow \text{HCF is } 12$$
$$= \frac{{}^{3}\cancel{9} \times b \times \cancel{(b-a)}^{1}}{{}_{4}\cancel{12}\cancel{(b-a)}_{1}}$$
$$= \frac{3b}{4}$$

4 **a**
$$\frac{2x-2y}{y-x}$$
$$= \frac{2\cancel{(x-y)}^{1}}{-1\cancel{(x-y)}_{1}}$$
$$= -2$$

b
$$\frac{3x-3y}{2y-2x}$$
$$= \frac{3\cancel{(x-y)}^{1}}{-2\cancel{(x-y)}_{1}}$$
$$= -\frac{3}{2}$$

c
$$\frac{m-n}{n-m}$$
$$= \frac{\cancel{(m-n)}^{1}}{-1\cancel{(m-n)}_{1}}$$
$$= -1$$

d
$$\frac{r-2s}{4s-2r}$$
$$= \frac{\cancel{(r-2s)}^{1}}{-2\cancel{(r-2s)}_{1}}$$
$$= -\frac{1}{2}$$

e $\dfrac{3r-6s}{2s-r}$

$= \dfrac{3(r-2s)^{1}}{-1(r-2s)_{1}}$

$= -3$

f $\dfrac{2x-2}{x-x^2}$

$= \dfrac{2(x-1)^{1}}{-x(x-1)_{1}}$

$= -\dfrac{2}{x}$

g $\dfrac{ab^2-ab}{2-2b}$

$= \dfrac{ab(b-1)^{1}}{-2(b-1)_{1}}$

$= -\dfrac{ab}{2}$

h $\dfrac{4x^2-4x}{2-2x}$

$= \dfrac{{}^{2}4x(x-1)^{1}}{{}_{-1}2(x-1)_{1}}$

$= -2x$

5 **a** $\dfrac{x^2-1}{x-1}$

$= \dfrac{(x+1)(x-1)^{1}}{(x-1)_{1}}$

$= x+1$

b $\dfrac{x^2-1}{x+1}$

$= \dfrac{{}^{1}(x+1)(x-1)}{{}_{1}(x+1)}$

$= x-1$

c $\dfrac{x^2-1}{1-x}$

$= \dfrac{(x+1)(x-1)^{1}}{-1(x-1)_{1}}$

$= -(x+1)$

$= -x-1$

d $\dfrac{x+2}{x^2-4}$

$= \dfrac{(x+2)^{1}}{{}_{1}(x+2)(x-2)}$

$= \dfrac{1}{x-2}$

e $\dfrac{a^2-b^2}{a+b}$

$= \dfrac{{}^{1}(a+b)(a-b)}{{}_{1}(a+b)}$

$= a-b$

f $\dfrac{a^2-b^2}{b-a}$

$= \dfrac{(a+b)(a-b)^{1}}{-1(a-b)_{1}}$

$= -(a+b)$

$= -a-b$

g $\dfrac{2x+2}{x^2-1}$

$= \dfrac{2(x+1)^{1}}{{}_{1}(x+1)(x-1)}$

$= \dfrac{2}{x-1}$

h $\dfrac{9-x^2}{3x-x^2}$

$= \dfrac{(3+x)(3-x)^{1}}{x(3-x)_{1}}$

$= \dfrac{3+x}{x}$

i $\dfrac{3x^2-3y^2}{2xy-2y^2}$

$= \dfrac{3(x^2-y^2)}{2y(x-y)}$

$= \dfrac{3(x+y)(x-y)^{1}}{2y(x-y)_{1}}$

$= \dfrac{3(x+y)}{2y}$

j $\dfrac{2b^2-2a^2}{a^2-ab}$

$= \dfrac{2(b^2-a^2)}{a(a-b)}$

$= \dfrac{2(b+a)(b-a)^{1}}{-a(b-a)_{1}}$

$= -\dfrac{2(b+a)}{a}$

k $\dfrac{4xy-y^2}{16x^2-y^2}$

$= \dfrac{y(4x-y)^{1}}{(4x+y)(4x-y)_{1}}$

$= \dfrac{y}{4x+y}$

l $\dfrac{4x(x-4)}{16-x^2}$

$= \dfrac{-4x(4-x)^{1}}{(4+x)(4-x)_{1}}$

$= \dfrac{-4x}{4+x}$

$= -\dfrac{4x}{x+4}$

6 **a** $\dfrac{x+4}{x^2+x-12}$

$= \dfrac{{}^{1}x+4}{{}_{1}(x+4)(x-3)}$

$= \dfrac{1}{x-3}$

b $\dfrac{x^2-x-2}{x-2}$

$= \dfrac{(x+1)(x-2)^{1}}{(x-2)_{1}}$

$= x+1$

c $\dfrac{x+3}{x^2-2x-15}$

$= \dfrac{(x+3)^{1}}{{}_{1}(x+3)(x-5)}$

$= \dfrac{1}{x-5}$

d $\dfrac{x^2-5x}{x^2+7x}$

$= \dfrac{{}^{1}x(x-5)}{{}_{1}x(x+7)}$

$= \dfrac{x-5}{x+7}$

e $\dfrac{2x^2+2x}{x^2-4x-5}$

$= \dfrac{2x(x+1)^{1}}{{}_{1}(x+1)(x-5)}$

$= \dfrac{2x}{x-5}$

f $\dfrac{x^2-4}{x^2+4x+4}$

$= \dfrac{{}^{1}(x+2)(x-2)}{{}_{1}(x+2)(x+2)}$

$= \dfrac{x-2}{x+2}$

g $\dfrac{x^2-x-12}{x^2-5x+4}$

$= \dfrac{(x+3)(x-4)^{1}}{(x-1)(x-4)_{1}}$

$= \dfrac{x+3}{x-1}$

h $\dfrac{x^2+2x+1}{1-x^2}$

$= \dfrac{{}^{1}(x+1)(x+1)}{{}_{1}(1+x)(1-x)}$

$\{\text{since } (x+1) = (1+x)\}$

$= \dfrac{x+1}{1-x}$

i $\dfrac{x^2-x-20}{x^2+7x+12}$

$= \dfrac{{}^{1}(x+4)(x-5)}{(x+3)(x+4)_{1}}$

$= \dfrac{x-5}{x+3}$

EXERCISE 4C

1 **a**
$$\frac{x}{2} \times \frac{y}{5} = \frac{x \times y}{2 \times 5} = \frac{xy}{10}$$

b
$$\frac{a}{2} \times \frac{3}{a} = \frac{{}^{1}\cancel{a} \times 3}{2 \times \cancel{a}_{1}} = \frac{3}{2}$$

c
$$\frac{a}{2} \times a = \frac{a}{2} \times \frac{a}{1} = \frac{a \times a}{2 \times 1} = \frac{a^2}{2}$$

d
$$\frac{a}{4} \times \frac{2}{3a} = \frac{a \times 2}{4 \times 3a} = \frac{{}^{1}\cancel{a} \times \cancel{2}^{1}}{{}_{2}\cancel{4} \times 3 \times \cancel{a}_{1}} = \frac{1}{6}$$

e
$$\frac{c}{5} \times \frac{1}{c} = \frac{{}^{1}\cancel{c} \times 1}{5 \times \cancel{c}_{1}} = \frac{1}{5}$$

f
$$\frac{c}{5} \times \frac{c}{2} = \frac{c \times c}{5 \times 2} = \frac{c^2}{10}$$

g
$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

h
$$\frac{a}{b} \times \frac{b}{a} = \frac{{}^{1}\cancel{a} \times \cancel{b}^{1}}{{}_{1}\cancel{b} \times \cancel{a}_{1}} = 1$$

i
$$\frac{1}{m^2} \times \frac{m}{2} = \frac{1 \times m}{m^2 \times 2} = \frac{1 \times \cancel{m}^{1}}{{}_{1}\cancel{m} \times m \times 2} = \frac{1}{2m}$$

j
$$\frac{m}{2} \times \frac{4}{m} = \frac{{}^{1}\cancel{m} \times \cancel{4}^{2}}{{}_{1}\cancel{2} \times \cancel{m}_{1}} = 2$$

k
$$\frac{a}{x} \times \frac{x}{b} = \frac{a \times \cancel{x}^{1}}{{}_{1}\cancel{x} \times b} = \frac{a}{b}$$

l
$$m \times \frac{4}{m} = \frac{m}{1} \times \frac{4}{m} = \frac{{}^{1}\cancel{m} \times 4}{1 \times \cancel{m}_{1}} = 4$$

m
$$\frac{3}{m^2} \times m = \frac{3}{m^2} \times \frac{m}{1} = \frac{3 \times m}{m^2 \times 1} = \frac{3 \times \cancel{m}^{1}}{{}_{1}\cancel{m} \times m \times 1} = \frac{3}{m}$$

n
$$\left(\frac{a}{b}\right)^2 = \frac{a}{b} \times \frac{a}{b} = \frac{a \times a}{b \times b} = \frac{a^2}{b^2}$$

o
$$\left(\frac{2}{x}\right)^2 = \frac{2}{x} \times \frac{2}{x} = \frac{2 \times 2}{x \times x} = \frac{4}{x^2}$$

p
$$\frac{1}{a} \times \frac{a}{b} \times \frac{b}{c} = \frac{1 \times {}^{1}\cancel{a} \times \cancel{b}^{1}}{{}_{1}\cancel{a} \times \cancel{b}_{1} \times c} = \frac{1}{c}$$

2 **a**
$$\frac{a}{2} \div \frac{a}{3} = \frac{a}{2} \times \frac{3}{a} = \frac{{}^{1}\cancel{a} \times 3}{2 \times \cancel{a}_{1}} = \frac{3}{2}$$

b
$$\frac{2}{a} \div \frac{2}{3} = \frac{2}{a} \times \frac{3}{2} = \frac{{}^{1}\cancel{2} \times 3}{a \times \cancel{2}_{1}} = \frac{3}{a}$$

c
$$\frac{3}{4} \div \frac{4}{x} = \frac{3}{4} \times \frac{x}{4} = \frac{3 \times x}{4 \times 4} = \frac{3x}{16}$$

d
$$\frac{3}{x} \div \frac{4}{x} = \frac{3}{x} \times \frac{x}{4} = \frac{3 \times \cancel{x}^{1}}{{}_{1}\cancel{x} \times 4} = \frac{3}{4}$$

e
$$\frac{2}{n} \div \frac{1}{n} = \frac{2}{n} \times \frac{n}{1} = \frac{2 \times \cancel{n}^{1}}{{}_{1}\cancel{n} \times 1} = 2$$

f
$$\frac{c}{5} \div 5 = \frac{c}{5} \div \frac{5}{1} = \frac{c}{5} \times \frac{1}{5} = \frac{c \times 1}{5 \times 5} = \frac{c}{25}$$

g
$$\frac{c}{5} \div c = \frac{c}{5} \div \frac{c}{1} = \frac{c}{5} \times \frac{1}{c} = \frac{{}^{1}\cancel{c} \times 1}{5 \times \cancel{c}_{1}} = \frac{1}{5}$$

h
$$m \div \frac{2}{m} = \frac{m}{1} \div \frac{2}{m} = \frac{m}{1} \times \frac{m}{2} = \frac{m \times m}{1 \times 2} = \frac{m^2}{2}$$

i
$$\begin{aligned} & m \div \frac{m}{2} \\ &= \frac{m}{1} \div \frac{m}{2} \\ &= \frac{m}{1} \times \frac{2}{m} \\ &= \frac{{}^{1}\cancel{m} \times 2}{1 \times \cancel{m}_{1}} \\ &= 2 \end{aligned}$$

j
$$\begin{aligned} & 1 \div \frac{m}{n} \\ &= \frac{1}{1} \div \frac{m}{n} \\ &= \frac{1}{1} \times \frac{n}{m} \\ &= \frac{1 \times n}{1 \times m} \\ &= \frac{n}{m} \end{aligned}$$

k
$$\begin{aligned} & \frac{3}{g} \div 4 \\ &= \frac{3}{g} \div \frac{4}{1} \\ &= \frac{3}{g} \times \frac{1}{4} \\ &= \frac{3 \times 1}{g \times 4} \\ &= \frac{3}{4g} \end{aligned}$$

l
$$\begin{aligned} & \frac{3}{g} \div \frac{9}{g^2} \\ &= \frac{3}{g} \times \frac{g^2}{9} \\ &= \frac{3 \times g^2}{g \times 9} \\ &= \frac{{}^{1}\cancel{3} \times g \times \cancel{g}^{1}}{{}_{1}\cancel{g} \times \cancel{9}_{3}} \\ &= \frac{g}{3} \end{aligned}$$

3 **a**
$$\begin{aligned} & \frac{x+1}{x^2-2x} \times \frac{x}{x^2-1} \\ &= \frac{{}^{1}\cancel{x+1}}{{}_{1}\cancel{x}(x-2)} \times \frac{{}^{1}\cancel{x}}{{}_{1}\cancel{(x+1)}(x-1)} \\ &= \frac{1}{(x-2)(x-1)} \end{aligned}$$

b
$$\begin{aligned} & \frac{x-2}{3} \div \frac{4-2x}{5} \\ &= \frac{x-2}{3} \times \frac{5}{4-2x} \\ &= \frac{x-2}{3} \times \frac{5}{2(2-x)} \\ &= \frac{{}^{1}\cancel{x-2}}{3} \times \frac{5}{-2\cancel{(x-2)}_{1}} \\ &= -\frac{5}{6} \end{aligned}$$

c
$$\begin{aligned} & \frac{x^2+3x}{x-2} \times \frac{5}{2x+6} \\ &= \frac{x\cancel{(x+3)}^{1}}{x-2} \times \frac{5}{2\cancel{(x+3)}_{1}} \\ &= \frac{5x}{2(x-2)} \end{aligned}$$

d
$$\begin{aligned} & \frac{t-5}{t^2+t} \times \frac{4t+4}{3t-15} \\ &= \frac{\cancel{t-5}^{1}}{t\cancel{(t+1)}_{1}} \times \frac{4\cancel{(t+1)}^{1}}{3\cancel{(t-5)}_{1}} \\ &= \frac{4}{3t} \end{aligned}$$

e
$$\begin{aligned} & \frac{4a-28}{a} \div \frac{a-7}{5} \\ &= \frac{4a-28}{a} \times \frac{5}{a-7} \\ &= \frac{4\cancel{(a-7)}^{1}}{a} \times \frac{5}{\cancel{a-7}_{1}} \\ &= \frac{20}{a} \end{aligned}$$

f
$$\begin{aligned} & \frac{6k-2}{k+2} \times \frac{2k^2+4k}{9k-3} \\ &= \frac{2\cancel{(3k-1)}^{1}}{\cancel{k+2}_{1}} \times \frac{2k\cancel{(k+2)}^{1}}{3\cancel{(3k-1)}_{1}} \\ &= \frac{4k}{3} \end{aligned}$$

EXERCISE 4D

1 **a**
$$\begin{aligned} & \frac{a}{2} + \frac{a}{3} \quad \{\text{LCD} = 6\} \\ &= \frac{a \times \mathbf{3}}{2 \times \mathbf{3}} + \frac{a \times \mathbf{2}}{3 \times \mathbf{2}} \\ &= \frac{3a}{6} + \frac{2a}{6} \\ &= \frac{3a+2a}{6} \\ &= \frac{5a}{6} \end{aligned}$$

b
$$\begin{aligned} & \frac{b}{5} - \frac{b}{10} \quad \{\text{LCD} = 10\} \\ &= \frac{b \times \mathbf{2}}{5 \times \mathbf{2}} - \frac{b}{10} \\ &= \frac{2b}{10} - \frac{b}{10} \\ &= \frac{2b-b}{10} \\ &= \frac{b}{10} \end{aligned}$$

c
$$\begin{aligned} & \frac{c}{4} + \frac{3c}{2} \quad \{\text{LCD} = 4\} \\ &= \frac{c}{4} + \frac{3c \times \mathbf{2}}{2 \times \mathbf{2}} \\ &= \frac{c}{4} + \frac{6c}{4} \\ &= \frac{c+6c}{4} \\ &= \frac{7c}{4} \end{aligned}$$

d
$$\begin{aligned} & \frac{d}{2} - \frac{3}{5} \quad \{\text{LCD} = 10\} \\ &= \frac{d \times \mathbf{5}}{2 \times \mathbf{5}} - \frac{3 \times \mathbf{2}}{5 \times \mathbf{2}} \\ &= \frac{5d}{10} - \frac{6}{10} \\ &= \frac{5d-6}{10} \end{aligned}$$

e
$$\begin{aligned} & \frac{5}{8} + \frac{x}{12} \quad \{\text{LCD} = 24\} \\ &= \frac{5 \times \mathbf{3}}{8 \times \mathbf{3}} + \frac{x \times \mathbf{2}}{12 \times \mathbf{2}} \\ &= \frac{15}{24} + \frac{2x}{24} \\ &= \frac{15+2x}{24} \\ &= \frac{2x+15}{24} \end{aligned}$$

f
$$\begin{aligned} & \frac{x}{7} - \frac{x}{2} \quad \{\text{LCD} = 14\} \\ &= \frac{x \times \mathbf{2}}{7 \times \mathbf{2}} - \frac{x \times \mathbf{7}}{2 \times \mathbf{7}} \\ &= \frac{2x}{14} - \frac{7x}{14} \\ &= \frac{2x-7x}{14} \\ &= \frac{-5x}{14} = -\frac{5x}{14} \end{aligned}$$

g $\frac{a}{3} + \frac{b}{4}$ $\{\text{LCD} = 12\}$

$= \frac{a \times \mathbf{4}}{3 \times \mathbf{4}} + \frac{b \times \mathbf{3}}{4 \times \mathbf{3}}$

$= \frac{4a}{12} + \frac{3b}{12}$

$= \frac{4a + 3b}{12}$

h $\frac{t}{3} - \frac{5t}{9}$ $\{\text{LCD} = 9\}$

$= \frac{t \times \mathbf{3}}{3 \times \mathbf{3}} - \frac{5t}{9}$

$= \frac{3t}{9} - \frac{5t}{9}$

$= \frac{3t - 5t}{9}$

$= \frac{-2t}{9} = -\frac{2t}{9}$

2 **a** $\frac{m}{7} + \frac{2m}{21}$ $\{\text{LCD} = 21\}$

$= \frac{m \times \mathbf{3}}{7 \times \mathbf{3}} + \frac{2m}{21}$

$= \frac{3m}{21} + \frac{2m}{21}$

$= \frac{3m + 2m}{21}$

$= \frac{5m}{21}$

b $\frac{5d}{6} - \frac{d}{3}$ $\{\text{LCD} = 6\}$

$= \frac{5d}{6} - \frac{d \times \mathbf{2}}{3 \times \mathbf{2}}$

$= \frac{5d}{6} - \frac{2d}{6}$

$= \frac{5d - 2d}{6}$

$= \frac{{}^{1}\cancel{3}d}{{}_{2}\cancel{6}} = \frac{d}{2}$

c $\frac{3p}{5} - \frac{2p}{7}$ $\{\text{LCD} = 35\}$

$= \frac{3p \times \mathbf{7}}{5 \times \mathbf{7}} - \frac{2p \times \mathbf{5}}{7 \times \mathbf{5}}$

$= \frac{21p}{35} - \frac{10p}{35}$

$= \frac{21p - 10p}{35}$

$= \frac{11p}{35}$

d $\frac{2t}{9} + \frac{4t}{15}$ $\{\text{LCD} = 45\}$

$= \frac{2t \times \mathbf{5}}{9 \times \mathbf{5}} + \frac{4t \times \mathbf{3}}{15 \times \mathbf{3}}$

$= \frac{10t}{45} + \frac{12t}{45}$

$= \frac{10t + 12t}{45}$

$= \frac{22t}{45}$

e $\frac{7k}{8} - \frac{11k}{18}$ $\{\text{LCD} = 72\}$

$= \frac{7k \times \mathbf{9}}{8 \times \mathbf{9}} - \frac{11k \times \mathbf{4}}{18 \times \mathbf{4}}$

$= \frac{63k}{72} - \frac{44k}{72}$

$= \frac{63k - 44k}{72}$

$= \frac{19k}{72}$

f $\frac{m}{2} + \frac{m}{3} + \frac{m}{6}$ $\{\text{LCD} = 6\}$

$= \frac{m \times \mathbf{3}}{2 \times \mathbf{3}} + \frac{m \times \mathbf{2}}{3 \times \mathbf{2}} + \frac{m}{6}$

$= \frac{3m}{6} + \frac{2m}{6} + \frac{m}{6}$

$= \frac{3m + 2m + m}{6}$

$= \frac{{}^{1}\cancel{6}m}{{}_{1}\cancel{6}}$

$= m$

g $\frac{a}{2} - \frac{a}{3} + \frac{a}{4}$ $\{\text{LCD} = 12\}$

$= \frac{a \times \mathbf{6}}{2 \times \mathbf{6}} - \frac{a \times \mathbf{4}}{3 \times \mathbf{4}} + \frac{a \times \mathbf{3}}{4 \times \mathbf{3}}$

$= \frac{6a}{12} - \frac{4a}{12} + \frac{3a}{12}$

$= \frac{6a - 4a + 3a}{12}$

$= \frac{5a}{12}$

h $\frac{x}{4} - \frac{x}{3} + \frac{x}{6}$ $\{\text{LCD} = 12\}$

$= \frac{x \times \mathbf{3}}{4 \times \mathbf{3}} - \frac{x \times \mathbf{4}}{3 \times \mathbf{4}} + \frac{x \times \mathbf{2}}{6 \times \mathbf{2}}$

$= \frac{3x}{12} - \frac{4x}{12} + \frac{2x}{12}$

$= \frac{3x - 4x + 2x}{12}$

$= \frac{x}{12}$

3 **a** $\frac{7}{a}+\frac{3}{b}$ $\{\text{LCD}=ab\}$

$=\frac{7\times \boldsymbol{b}}{a\times \boldsymbol{b}}+\frac{3\times \boldsymbol{a}}{b\times \boldsymbol{a}}$

$=\frac{7b}{ab}+\frac{3a}{ab}$

$=\frac{7b+3a}{ab}$

b $\frac{3}{a}+\frac{2}{c}$ $\{\text{LCD}=ac\}$

$=\frac{3\times \boldsymbol{c}}{a\times \boldsymbol{c}}+\frac{2\times \boldsymbol{a}}{c\times \boldsymbol{a}}$

$=\frac{3c}{ac}+\frac{2a}{ac}$

$=\frac{3c+2a}{ac}$

c $\frac{4}{a}+\frac{5}{d}$ $\{\text{LCD}=ad\}$

$=\frac{4\times \boldsymbol{d}}{a\times \boldsymbol{d}}+\frac{5\times \boldsymbol{a}}{d\times \boldsymbol{a}}$

$=\frac{4d}{ad}+\frac{5a}{ad}$

$=\frac{4d+5a}{ad}$

d $\frac{2a}{m}-\frac{a}{n}$ $\{\text{LCD}=mn\}$

$=\frac{2a\times \boldsymbol{n}}{m\times \boldsymbol{n}}-\frac{a\times \boldsymbol{m}}{n\times \boldsymbol{m}}$

$=\frac{2an}{mn}-\frac{am}{mn}$

$=\frac{2an-am}{mn}$

e $\frac{a}{x}+\frac{b}{2x}$ $\{\text{LCD}=2x\}$

$=\frac{a\times \boldsymbol{2}}{x\times \boldsymbol{2}}+\frac{b}{2x}$

$=\frac{2a}{2x}+\frac{b}{2x}$

$=\frac{2a+b}{2x}$

f $\frac{3}{a}-\frac{1}{2a}$ $\{\text{LCD}=2a\}$

$=\frac{3\times \boldsymbol{2}}{a\times \boldsymbol{2}}-\frac{1}{2a}$

$=\frac{6}{2a}-\frac{1}{2a}$

$=\frac{6-1}{2a}=\frac{5}{2a}$

g $\frac{4}{x}-\frac{1}{xy}$ $\{\text{LCD}=xy\}$

$=\frac{4\times \boldsymbol{y}}{x\times \boldsymbol{y}}-\frac{1}{xy}$

$=\frac{4y}{xy}-\frac{1}{xy}$

$=\frac{4y-1}{xy}$

h $\frac{5}{x}+\frac{6}{5x}$ $\{\text{LCD}=5x\}$

$=\frac{5\times \boldsymbol{5}}{x\times \boldsymbol{5}}+\frac{6}{5x}$

$=\frac{25}{5x}+\frac{6}{5x}$

$=\frac{25+6}{5x}$

$=\frac{31}{5x}$

i $\frac{11}{3z}-\frac{3}{4z}$ $\{\text{LCD}=12z\}$

$=\frac{11\times \boldsymbol{4}}{3z\times \boldsymbol{4}}-\frac{3\times \boldsymbol{3}}{4z\times \boldsymbol{3}}$

$=\frac{44}{12z}-\frac{9}{12z}$

$=\frac{44-9}{12z}$

$=\frac{35}{12z}$

j $\frac{a}{b}+\frac{c}{d}$ $\{\text{LCD}=bd\}$

$=\frac{a\times \boldsymbol{d}}{b\times \boldsymbol{d}}+\frac{c\times \boldsymbol{b}}{d\times \boldsymbol{b}}$

$=\frac{ad}{bd}+\frac{bc}{bd}$

$=\frac{ad+bc}{bd}$

k $\frac{3}{a}+\frac{a}{2}$ $\{\text{LCD}=2a\}$

$=\frac{3\times \boldsymbol{2}}{a\times \boldsymbol{2}}+\frac{a\times \boldsymbol{a}}{2\times \boldsymbol{a}}$

$=\frac{6}{2a}+\frac{a^2}{2a}$

$=\frac{6+a^2}{2a}$

l $\frac{x}{y}+\frac{2}{3}$ $\{\text{LCD}=3y\}$

$=\frac{x\times \boldsymbol{3}}{y\times \boldsymbol{3}}+\frac{2\times \boldsymbol{y}}{3\times \boldsymbol{y}}$

$=\frac{3x}{3y}+\frac{2y}{3y}$

$=\frac{3x+2y}{3y}$

m $\frac{8}{p}-\frac{2}{5}$ $\{\text{LCD}=5p\}$

$=\frac{8\times \boldsymbol{5}}{p\times \boldsymbol{5}}-\frac{2\times \boldsymbol{p}}{5\times \boldsymbol{p}}$

$=\frac{40}{5p}-\frac{2p}{5p}$

$=\frac{40-2p}{5p}$

n $\frac{x}{6y}+\frac{2x}{9y}$ $\{\text{LCD}=18y\}$

$=\frac{x\times \boldsymbol{3}}{6y\times \boldsymbol{3}}+\frac{2x\times \boldsymbol{2}}{9y\times \boldsymbol{2}}$

$=\frac{3x}{18y}+\frac{4x}{18y}$

$=\frac{3x+4x}{18y}$

$=\frac{7x}{18y}$

o $\frac{1}{8t}-\frac{3}{5t}$ $\{\text{LCD}=40t\}$

$=\frac{1\times \boldsymbol{5}}{8t\times \boldsymbol{5}}-\frac{3\times \boldsymbol{8}}{5t\times \boldsymbol{8}}$

$=\frac{5}{40t}-\frac{24}{40t}$

$=\frac{5-24}{40t}$

$=\frac{-19}{40t}=-\frac{19}{40t}$

p $\frac{5}{2x}+\frac{3}{x^2}$ $\{\text{LCD}=2x^2\}$

$=\frac{5\times \boldsymbol{x}}{2x\times \boldsymbol{x}}+\frac{3\times \boldsymbol{2}}{x^2\times \boldsymbol{2}}$

$=\frac{5x}{2x^2}+\frac{6}{2x^2}$

$=\frac{5x+6}{2x^2}$

4 **a**
$$\begin{aligned}&\frac{x}{2}+1\\&=\frac{x}{2}+\frac{1\times\mathbf{2}}{1\times\mathbf{2}}\\&=\frac{x}{2}+\frac{2}{2}\\&=\frac{x+2}{2}\end{aligned}$$

b
$$\begin{aligned}&\frac{y}{3}-1\\&=\frac{y}{3}-\frac{1\times\mathbf{3}}{1\times\mathbf{3}}\\&=\frac{y}{3}-\frac{3}{3}\\&=\frac{y-3}{3}\end{aligned}$$

c
$$\begin{aligned}&\frac{a}{2}+a\\&=\frac{a}{2}+\frac{a\times\mathbf{2}}{1\times\mathbf{2}}\\&=\frac{a}{2}+\frac{2a}{2}\\&=\frac{a+2a}{2}\\&=\frac{3a}{2}\end{aligned}$$

d
$$\begin{aligned}&\frac{b}{4}-3\\&=\frac{b}{4}-\frac{3\times\mathbf{4}}{1\times\mathbf{4}}\\&=\frac{b}{4}-\frac{12}{4}\\&=\frac{b-12}{4}\end{aligned}$$

e
$$\begin{aligned}&\frac{x}{2}-4\\&=\frac{x}{2}-\frac{4\times\mathbf{2}}{1\times\mathbf{2}}\\&=\frac{x}{2}-\frac{8}{2}\\&=\frac{x-8}{2}\end{aligned}$$

f
$$\begin{aligned}&2+\frac{a}{3}\\&=\frac{2\times\mathbf{3}}{1\times\mathbf{3}}+\frac{a}{3}\\&=\frac{6}{3}+\frac{a}{3}\\&=\frac{6+a}{3}\end{aligned}$$

g
$$\begin{aligned}&x-\frac{x}{5}\\&=\frac{x\times\mathbf{5}}{1\times\mathbf{5}}-\frac{x}{5}\\&=\frac{5x}{5}-\frac{x}{5}\\&=\frac{5x-x}{5}\\&=\frac{4x}{5}\end{aligned}$$

h
$$\begin{aligned}&2+\frac{1}{x}\\&=\frac{2\times\boldsymbol{x}}{1\times\boldsymbol{x}}+\frac{1}{x}\\&=\frac{2x}{x}+\frac{1}{x}\\&=\frac{2x+1}{x}\end{aligned}$$

i
$$\begin{aligned}&5-\frac{2}{x}\\&=\frac{5\times\boldsymbol{x}}{1\times\boldsymbol{x}}-\frac{2}{x}\\&=\frac{5x}{x}-\frac{2}{x}\\&=\frac{5x-2}{x}\end{aligned}$$

j
$$\begin{aligned}&a+\frac{2}{a}\\&=\frac{a\times\boldsymbol{a}}{1\times\boldsymbol{a}}+\frac{2}{a}\\&=\frac{a^2}{a}+\frac{2}{a}\\&=\frac{a^2+2}{a}\end{aligned}$$

k
$$\begin{aligned}&\frac{3}{b}+b\\&=\frac{3}{b}+\frac{b\times\boldsymbol{b}}{1\times\boldsymbol{b}}\\&=\frac{3}{b}+\frac{b^2}{b}\\&=\frac{3+b^2}{b}\end{aligned}$$

l
$$\begin{aligned}&\frac{1}{x^2}-2x\\&=\frac{1}{x^2}-\frac{2x\times\boldsymbol{x^2}}{1\times\boldsymbol{x^2}}\\&=\frac{1}{x^2}-\frac{2x^3}{x^2}\\&=\frac{1-2x^3}{x^2}\end{aligned}$$

5 **a**
$$\begin{aligned}&\frac{x}{2}+\frac{x+1}{3}\quad\{\text{LCD}=6\}\\&=\frac{\mathbf{3}}{\mathbf{3}}\left(\frac{x}{2}\right)+\frac{\mathbf{2}}{\mathbf{2}}\left(\frac{x+1}{3}\right)\\&=\frac{3x}{6}+\frac{2(x+1)}{6}\\&=\frac{3x+2(x+1)}{6}\\&=\frac{3x+2x+2}{6}\\&=\frac{5x+2}{6}\end{aligned}$$

b
$$\begin{aligned}&\frac{x-1}{4}-\frac{x}{2}\quad\{\text{LCD}=4\}\\&=\frac{x-1}{4}-\frac{\mathbf{2}}{\mathbf{2}}\left(\frac{x}{2}\right)\\&=\frac{x-1}{4}-\frac{2x}{4}\\&=\frac{x-1-2x}{4}\\&=\frac{-x-1}{4}\end{aligned}$$

c
$$\begin{aligned}&\frac{2x}{3}+\frac{x+3}{4}\quad\{\text{LCD}=12\}\\&=\frac{\mathbf{4}}{\mathbf{4}}\left(\frac{2x}{3}\right)+\frac{\mathbf{3}}{\mathbf{3}}\left(\frac{x+3}{4}\right)\\&=\frac{8x}{12}+\frac{3(x+3)}{12}\\&=\frac{8x+3(x+3)}{12}\\&=\frac{8x+3x+9}{12}\\&=\frac{11x+9}{12}\end{aligned}$$

d
$$\begin{aligned}&\frac{x+1}{2}+\frac{x-1}{3}\quad\{\text{LCD}=6\}\\&=\frac{\mathbf{3}}{\mathbf{3}}\left(\frac{x+1}{2}\right)+\frac{\mathbf{2}}{\mathbf{2}}\left(\frac{x-1}{3}\right)\\&=\frac{3(x+1)}{6}+\frac{2(x-1)}{6}\\&=\frac{3(x+1)+2(x-1)}{6}\\&=\frac{3x+3+2x-2}{6}\\&=\frac{5x+1}{6}\end{aligned}$$

e $\dfrac{x-1}{3}+\dfrac{1-2x}{4}$ $\{\text{LCD}=12\}$

$=\dfrac{\mathbf{4}}{\mathbf{4}}\left(\dfrac{x-1}{3}\right)+\dfrac{\mathbf{3}}{\mathbf{3}}\left(\dfrac{1-2x}{4}\right)$

$=\dfrac{4(x-1)}{12}+\dfrac{3(1-2x)}{12}$

$=\dfrac{4(x-1)+3(1-2x)}{12}$

$=\dfrac{4x-4+3-6x}{12}$

$=\dfrac{-2x-1}{12}$

f $\dfrac{x}{3}+\dfrac{x+1}{4}$ $\{\text{LCD}=12\}$

$=\dfrac{\mathbf{4}}{\mathbf{4}}\left(\dfrac{x}{3}\right)+\dfrac{\mathbf{3}}{\mathbf{3}}\left(\dfrac{x+1}{4}\right)$

$=\dfrac{4x}{12}+\dfrac{3(x+1)}{12}$

$=\dfrac{4x+3(x+1)}{12}$

$=\dfrac{4x+3x+3}{12}$

$=\dfrac{7x+3}{12}$

g $\dfrac{3x+2}{4}+\dfrac{x}{2}$ $\{\text{LCD}=4\}$

$=\dfrac{3x+2}{4}+\dfrac{\mathbf{2}}{\mathbf{2}}\left(\dfrac{x}{2}\right)$

$=\dfrac{3x+2}{4}+\dfrac{2x}{4}$

$=\dfrac{3x+2+2x}{4}$

$=\dfrac{5x+2}{4}$

h $\dfrac{a+b}{3}+\dfrac{b-a}{2}$ $\{\text{LCD}=6\}$

$=\dfrac{\mathbf{2}}{\mathbf{2}}\left(\dfrac{a+b}{3}\right)+\dfrac{\mathbf{3}}{\mathbf{3}}\left(\dfrac{b-a}{2}\right)$

$=\dfrac{2(a+b)}{6}+\dfrac{3(b-a)}{6}$

$=\dfrac{2(a+b)+3(b-a)}{6}$

$=\dfrac{2a+2b+3b-3a}{6}$

$=\dfrac{5b-a}{6}$

i $\dfrac{x+1}{5}+\dfrac{2x-1}{4}$ $\{\text{LCD}=20\}$

$=\dfrac{\mathbf{4}}{\mathbf{4}}\left(\dfrac{x+1}{5}\right)+\dfrac{\mathbf{5}}{\mathbf{5}}\left(\dfrac{2x-1}{4}\right)$

$=\dfrac{4(x+1)}{20}+\dfrac{5(2x-1)}{20}$

$=\dfrac{4(x+1)+5(2x-1)}{20}$

$=\dfrac{4x+4+10x-5}{20}$

$=\dfrac{14x-1}{20}$

j $\dfrac{x+1}{7}+\dfrac{3-x}{2}$ $\{\text{LCD}=14\}$

$=\dfrac{\mathbf{2}}{\mathbf{2}}\left(\dfrac{x+1}{7}\right)+\dfrac{\mathbf{7}}{\mathbf{7}}\left(\dfrac{3-x}{2}\right)$

$=\dfrac{2(x+1)}{14}+\dfrac{7(3-x)}{14}$

$=\dfrac{2(x+1)+7(3-x)}{14}$

$=\dfrac{2x+2+21-7x}{14}$

$=\dfrac{23-5x}{14}$

k $\dfrac{x}{6}-\dfrac{2-x}{5}$ $\{\text{LCD}=30\}$

$=\dfrac{\mathbf{5}}{\mathbf{5}}\left(\dfrac{x}{6}\right)-\dfrac{\mathbf{6}}{\mathbf{6}}\left(\dfrac{2-x}{5}\right)$

$=\dfrac{5x}{30}-\dfrac{6(2-x)}{30}$

$=\dfrac{5x-6(2-x)}{30}$

$=\dfrac{5x-12+6x}{30}$

$=\dfrac{11x-12}{30}$

l $\dfrac{2x-1}{5}-\dfrac{x}{4}$ $\{\text{LCD}=20\}$

$=\dfrac{\mathbf{4}}{\mathbf{4}}\left(\dfrac{2x-1}{5}\right)-\dfrac{\mathbf{5}}{\mathbf{5}}\left(\dfrac{x}{4}\right)$

$=\dfrac{4(2x-1)}{20}-\dfrac{5x}{20}$

$=\dfrac{4(2x-1)-5x}{20}$

$=\dfrac{8x-4-5x}{20}$

$=\dfrac{3x-4}{20}$

m $\dfrac{x}{8} - \dfrac{1-x}{4}$ $\{\text{LCD} = 8\}$

$= \dfrac{x}{8} - \dfrac{\mathbf{2}}{\mathbf{2}}\left(\dfrac{1-x}{4}\right)$

$= \dfrac{x}{8} - \dfrac{2(1-x)}{8}$

$= \dfrac{x - 2(1-x)}{8}$

$= \dfrac{x - 2 + 2x}{8}$

$= \dfrac{3x - 2}{8}$

n $\dfrac{x-1}{5} - \dfrac{2x-7}{3}$ $\{\text{LCD} = 15\}$

$= \dfrac{\mathbf{3}}{\mathbf{3}}\left(\dfrac{x-1}{5}\right) - \dfrac{\mathbf{5}}{\mathbf{5}}\left(\dfrac{2x-7}{3}\right)$

$= \dfrac{3(x-1)}{15} - \dfrac{5(2x-7)}{15}$

$= \dfrac{3(x-1) - 5(2x-7)}{15}$

$= \dfrac{3x - 3 - 10x + 35}{15}$

$= \dfrac{32 - 7x}{15}$

o $\dfrac{1-3x}{4} - \dfrac{2x+1}{3}$ $\{\text{LCD} = 12\}$

$= \dfrac{\mathbf{3}}{\mathbf{3}}\left(\dfrac{1-3x}{4}\right) - \dfrac{\mathbf{4}}{\mathbf{4}}\left(\dfrac{2x+1}{3}\right)$

$= \dfrac{3(1-3x)}{12} - \dfrac{4(2x+1)}{12}$

$= \dfrac{3(1-3x) - 4(2x+1)}{12}$

$= \dfrac{3 - 9x - 8x - 4}{12}$

$= \dfrac{-17x - 1}{12}$

6 **a** $\dfrac{3}{x} + \dfrac{4}{x+1}$ $\{\text{LCD} = x(x+1)\}$

$= \dfrac{3}{x}\left(\dfrac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right) + \left(\dfrac{4}{x+1}\right)\dfrac{\boldsymbol{x}}{\boldsymbol{x}}$

$= \dfrac{3(x+1) + 4x}{x(x+1)}$

$= \dfrac{3x + 3 + 4x}{x(x+1)}$

$= \dfrac{7x + 3}{x(x+1)}$

b $\dfrac{5}{x+2} - \dfrac{3}{x}$ $\{\text{LCD} = x(x+2)\}$

$= \left(\dfrac{5}{x+2}\right)\dfrac{\boldsymbol{x}}{\boldsymbol{x}} - \dfrac{3}{x}\left(\dfrac{\boldsymbol{x+2}}{\boldsymbol{x+2}}\right)$

$= \dfrac{5x - 3(x+2)}{x(x+2)}$

$= \dfrac{5x - 3x - 6}{x(x+2)}$

$= \dfrac{2x - 6}{x(x+2)}$ or $\dfrac{2(x-3)}{x(x+2)}$

c $\dfrac{4}{x+1} - \dfrac{3}{x-1}$ $\{\text{LCD} = (x+1)(x-1)\}$

$= \dfrac{4}{x+1}\left(\dfrac{\boldsymbol{x-1}}{\boldsymbol{x-1}}\right) - \dfrac{3}{x-1}\left(\dfrac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right)$

$= \dfrac{4(x-1) - 3(x+1)}{(x+1)(x-1)}$

$= \dfrac{4x - 4 - 3x - 3}{(x+1)(x-1)}$

$= \dfrac{x - 7}{(x+1)(x-1)}$

d $3 + \dfrac{1}{x+2}$ $\{\text{LCD} = (x+2)\}$

$= \dfrac{3}{1}\left(\dfrac{\boldsymbol{x+2}}{\boldsymbol{x+2}}\right) + \dfrac{1}{x+2}$

$= \dfrac{3(x+2) + 1}{x+2}$

$= \dfrac{3x + 6 + 1}{x+2}$

$= \dfrac{3x + 7}{x+2}$

e $\frac{1}{x} + \frac{4}{x-4}$ $\{\text{LCD} = x(x-4)\}$

$= \frac{1}{x}\left(\frac{\boldsymbol{x-4}}{\boldsymbol{x-4}}\right) + \left(\frac{4}{x-4}\right)\frac{\boldsymbol{x}}{\boldsymbol{x}}$

$= \frac{(x-4)+4x}{x(x-4)}$

$= \frac{5x-4}{x(x-4)}$

f $\frac{2}{x+3} - 4$ $\{\text{LCD} = (x+3)\}$

$= \frac{2}{x+3} - \frac{4}{1}\left(\frac{\boldsymbol{x+3}}{\boldsymbol{x+3}}\right)$

$= \frac{2-4(x+3)}{x+3}$

$= \frac{2-4x-12}{x+3}$

$= \frac{-4x-10}{x+3}$ or $\frac{-2(2x+5)}{x+3}$

g $\frac{x+1}{x-1} + \frac{x}{x+1}$ $\{\text{LCD} = (x-1)(x+1)\}$

$= \left(\frac{x+1}{x-1}\right)\left(\frac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right) + \left(\frac{x}{x+1}\right)\left(\frac{\boldsymbol{x-1}}{\boldsymbol{x-1}}\right)$

$= \frac{(x+1)^2 + x(x-1)}{(x+1)(x-1)}$

$= \frac{x^2+2x+1+x^2-x}{(x+1)(x-1)}$

$= \frac{2x^2+x+1}{(x+1)(x-1)}$

h $\frac{5}{x} + \frac{6}{x-2}$ $\{\text{LCD} = x(x-2)\}$

$= \frac{5}{x}\left(\frac{\boldsymbol{x-2}}{\boldsymbol{x-2}}\right) + \left(\frac{6}{x-2}\right)\frac{\boldsymbol{x}}{\boldsymbol{x}}$

$= \frac{5(x-2)+6x}{x(x-2)}$

$= \frac{5x-10+6x}{x(x-2)}$

$= \frac{11x-10}{x(x-2)}$

i $\frac{x}{x+5} - \frac{x}{x-3}$ $\{\text{LCD} = (x+5)(x-3)\}$

$= \left(\frac{x}{x+5}\right)\left(\frac{\boldsymbol{x-3}}{\boldsymbol{x-3}}\right) - \left(\frac{x}{x-3}\right)\left(\frac{\boldsymbol{x+5}}{\boldsymbol{x+5}}\right)$

$= \frac{x(x-3)-x(x+5)}{(x+5)(x-3)}$

$= \frac{x^2-3x-x^2-5x}{(x+5)(x-3)}$

$= \frac{-8x}{(x+5)(x-3)}$

7 **a** Suppose there are x players in a team.

The \$150 court fee is shared between the players, so each player pays $\$\left(\frac{150}{x}\right)$.

The \$60 registration fee is shared between the players and coach, so each person pays $\$\left(\frac{60}{x+1}\right)$.

So, each player pays a total of $\frac{150}{x} + \frac{60}{x+1}$ dollars.

Now, $\frac{150}{x} + \frac{60}{x+1} = \frac{150}{x}\left(\frac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right) + \left(\frac{60}{x+1}\right)\frac{\boldsymbol{x}}{\boldsymbol{x}}$

$= \frac{150(x+1)+60x}{x(x+1)}$

$= \frac{150x+150+60x}{x(x+1)}$

$= \frac{210x+150}{x(x+1)}$ or $\frac{30(7x+5)}{x(x+1)}$

So, each player pays $\$\left(\frac{210x+150}{x(x+1)}\right)$.

b **i** If $x = 4$, then

$$\frac{210x+150}{x(x+1)} = \frac{210(4)+150}{4(4+1)}$$
$$= \frac{840+150}{4\times 5}$$
$$= \frac{990}{20}$$
$$= 49.5$$

If the team has 4 players, each player pays \$49.50.

ii If $x = 5$, then

$$\frac{210x+150}{x(x+1)} = \frac{210(5)+150}{5(5+1)}$$
$$= \frac{1050+150}{5\times 6}$$
$$= \frac{1200}{30}$$
$$= 40$$

If the team has 5 players, each player pays \$40.

REVIEW SET 4

1 **a**
$$\frac{r}{q} = \frac{6}{-3} = -2$$

b
$$\frac{p+2q}{r} = \frac{5+2\times(-3)}{6} = \frac{5-6}{6} = -\tfrac{1}{6}$$

c
$$\frac{p-q}{p+q} = \frac{5-(-3)}{5+(-3)} = \frac{5+3}{5-3} = \frac{8}{2} = 4$$

d
$$\frac{\sqrt{p^2-16}}{r-q} = \frac{\sqrt{5^2-16}}{6-(-3)} = \frac{\sqrt{25-16}}{6+3} = \frac{\sqrt{9}}{9} = \tfrac{3}{9} = \tfrac{1}{3}$$

2 **a**
$$\frac{(2t)^2}{6t} = \frac{{}^{1}\cancel{2t}\times 2t}{{}_{3}\cancel{6t}} = \frac{2t}{3}$$

b
$$\frac{16a+8b}{6a+3b} = \frac{8(2a+b)^{1}}{3(2a+b)_{1}} = \frac{8}{3}$$

c
$$\frac{x(x-4)}{3(x-4)} = \frac{x(x-4)^{1}}{3(x-4)_{1}} = \frac{x}{3}$$

d
$$\frac{8}{4x+8} = \frac{{}^{2}\cancel{8}}{{}_{1}\cancel{4}(x+2)} = \frac{2}{x+2}$$

3 **a**
$$\frac{2x+6}{x^2-9} = \frac{2(x+3)^{1}}{{}_{1}(x+3)(x-3)} = \frac{2}{x-3}$$

b
$$\frac{x^2+4x+4}{x^2+2x} = \frac{(x+2)(x+2)^{1}}{x(x+2)_{1}} = \frac{x+2}{x}$$

c
$$\frac{3x^2-6x}{x^2-x-2} = \frac{3x(x-2)^{1}}{{}_{1}(x-2)(x+1)} = \frac{3x}{x+1}$$

4 **a**
$$\frac{2a-2b}{b-a} = \frac{2(a-b)^{1}}{-1(a-b)_{1}} = \frac{2}{-1} = -2$$

b
$$\frac{5x-15}{3x-x^2} = \frac{5(x-3)^{1}}{-x(x-3)_{1}} = \frac{5}{-x} = -\frac{5}{x}$$

c
$$\frac{16-x^2}{2x-8} = \frac{(4+x)(4-x)^{1}}{-2(4-x)_{1}} = \frac{x+4}{-2} = -\frac{x+4}{2}$$

5 **a**
$$\frac{a}{b}\times\frac{b}{3} = \frac{a\times \cancel{b}^{1}}{{}_{1}\cancel{b}\times 3} = \frac{a}{3}$$

b
$$\frac{a}{b}\div\frac{b}{3} = \frac{a}{b}\times\frac{3}{b} = \frac{a\times 3}{b\times b} = \frac{3a}{b^2}$$

c $\{\text{LCD} = 3b\}$
$$\frac{a}{b}+\frac{b}{3} = \frac{a\times \mathbf{3}}{b\times \mathbf{3}} + \frac{b\times \boldsymbol{b}}{3\times \boldsymbol{b}} = \frac{3a}{3b} + \frac{b^2}{3b} = \frac{3a+b^2}{3b}$$

d $\{\text{LCD} = 3b\}$
$$\frac{a}{b}-\frac{b}{3} = \frac{a\times \mathbf{3}}{b\times \mathbf{3}} - \frac{b\times \boldsymbol{b}}{3\times \boldsymbol{b}} = \frac{3a}{3b} - \frac{b^2}{3b} = \frac{3a-b^2}{3b}$$

6 **a**
$$\frac{7x-14}{x} \times \frac{3}{x-2}$$
$$= \frac{7(x-2)}{x} \times \frac{3}{(x-2)}$$
$$= \frac{7 \times \cancel{(x-2)}^{1} \times 3}{x \times \cancel{(x-2)}_{1}}$$
$$= \frac{21}{x}$$

b
$$\frac{t^2-3t}{6t+6} \times \frac{t+1}{4t-12}$$
$$= \frac{t(t-3)}{6(t+1)} \times \frac{t+1}{4(t-3)}$$
$$= \frac{t \times^{1} \cancel{(t-3)} \times \cancel{(t+1)}^{1}}{6 \times_{1} \cancel{(t+1)} \times 4 \times \cancel{(t-3)}_{1}}$$
$$= \frac{t}{24}$$

7 **a**
$$\frac{9}{n} \div 6$$
$$= \frac{9}{n} \div \frac{6}{1}$$
$$= \frac{9}{n} \times \frac{1}{6}$$
$$= \frac{^{3}\cancel{9} \times 1}{n \times \cancel{6}_{2}}$$
$$= \frac{3}{2n}$$

b
$$\frac{3x-6}{2} \div \frac{x^2-2x}{4}$$
$$= \frac{3(x-2)}{2} \times \frac{4}{x^2-2x}$$
$$= \frac{3\cancel{(x-2)}^{1}}{_{1}\cancel{2}} \times \frac{^{2}\cancel{4}}{x\cancel{(x-2)}_{1}}$$
$$= \frac{6}{x}$$

8 **a**
$$\frac{2x}{3} + \frac{x}{4} \qquad \{\text{LCD} = 12\}$$
$$= \frac{2x \times \mathbf{4}}{3 \times \mathbf{4}} + \frac{x \times \mathbf{3}}{4 \times \mathbf{3}}$$
$$= \frac{8x}{12} + \frac{3x}{12}$$
$$= \frac{11x}{12}$$

b
$$2 + \frac{x}{7} \qquad \{\text{LCD} = 7\}$$
$$= \frac{2 \times \mathbf{7}}{1 \times \mathbf{7}} + \frac{x}{7}$$
$$= \frac{14}{7} + \frac{x}{7}$$
$$= \frac{14+x}{7}$$

c
$$\frac{x}{4} - 1 \qquad \{\text{LCD} = 4\}$$
$$= \frac{x}{4} - \frac{1 \times \mathbf{4}}{1 \times \mathbf{4}}$$
$$= \frac{x}{4} - \frac{4}{4}$$
$$= \frac{x-4}{4}$$

d
$$\frac{x}{2} + \frac{x}{4} - \frac{x}{3} \qquad \{\text{LCD} = 12\}$$
$$= \frac{x \times \mathbf{6}}{2 \times \mathbf{6}} + \frac{x \times \mathbf{3}}{4 \times \mathbf{3}} - \frac{x \times \mathbf{4}}{3 \times \mathbf{4}}$$
$$= \frac{6x}{12} + \frac{3x}{12} - \frac{4x}{12}$$
$$= \frac{5x}{12}$$

9 **a**
$$\frac{x}{3} + \frac{x-1}{4} \qquad \{\text{LCD} = 12\}$$
$$= \left(\frac{x}{3}\right)\frac{\mathbf{4}}{\mathbf{4}} + \left(\frac{x-1}{4}\right)\frac{\mathbf{3}}{\mathbf{3}}$$
$$= \frac{4x}{12} + \frac{3(x-1)}{12}$$
$$= \frac{4x+3(x-1)}{12}$$
$$= \frac{4x+3x-3}{12}$$
$$= \frac{7x-3}{12}$$

b
$$\frac{x+2}{3} - \frac{2-x}{6} \qquad \{\text{LCD} = 6\}$$
$$= \left(\frac{x+2}{3}\right)\frac{\mathbf{2}}{\mathbf{2}} - \frac{2-x}{6}$$
$$= \frac{2(x+2)}{6} - \frac{2-x}{6}$$
$$= \frac{2(x+2)-(2-x)}{6}$$
$$= \frac{2x+4-2+x}{6}$$
$$= \frac{3x+2}{6}$$

c $\dfrac{2x+1}{5} - \dfrac{x-1}{10}$ $\{\text{LCD} = 10\}$

$= \left(\dfrac{2x+1}{5}\right)\dfrac{\mathbf{2}}{\mathbf{2}} - \dfrac{x-1}{10}$

$= \dfrac{2(2x+1)}{10} - \dfrac{x-1}{10}$

$= \dfrac{2(2x+1)-(x-1)}{10}$

$= \dfrac{4x+2-x+1}{10}$

$= \dfrac{3x+3}{10}$

10 **a** $\dfrac{1}{x+1} + \dfrac{2}{x-2}$ $\{\text{LCD} = (x+1)(x-2)\}$

$= \left(\dfrac{1}{x+1}\right)\left(\dfrac{\boldsymbol{x-2}}{\boldsymbol{x-2}}\right) + \left(\dfrac{2}{x-2}\right)\left(\dfrac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right)$

$= \dfrac{x-2}{(x+1)(x-2)} + \dfrac{2(x+1)}{(x+1)(x-2)}$

$= \dfrac{x-2+2(x+1)}{(x+1)(x-2)}$

$= \dfrac{x-2+2x+2}{(x+1)(x-2)}$

$= \dfrac{3x}{(x+1)(x-2)}$

b $\dfrac{5}{x-1} - \dfrac{4}{x+1}$ $\{\text{LCD} = (x-1)(x+1)\}$

$= \left(\dfrac{5}{x-1}\right)\left(\dfrac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right) - \left(\dfrac{4}{x+1}\right)\left(\dfrac{\boldsymbol{x-1}}{\boldsymbol{x-1}}\right)$

$= \dfrac{5(x+1)}{(x-1)(x+1)} - \dfrac{4(x-1)}{(x-1)(x+1)}$

$= \dfrac{5(x+1)-4(x-1)}{(x-1)(x+1)}$

$= \dfrac{5x+5-4x+4}{(x-1)(x+1)}$

$= \dfrac{x+9}{(x-1)(x+1)}$

c $\dfrac{1}{x^2} + \dfrac{1}{x+1}$ $\{\text{LCD} = x^2(x+1)\}$

$= \left(\dfrac{1}{x^2}\right)\left(\dfrac{\boldsymbol{x+1}}{\boldsymbol{x+1}}\right) + \left(\dfrac{1}{x+1}\right)\dfrac{\boldsymbol{x^2}}{\boldsymbol{x^2}}$

$= \dfrac{x+1}{x^2(x+1)} + \dfrac{x^2}{x^2(x+1)}$

$= \dfrac{x+1+x^2}{x^2(x+1)}$

$= \dfrac{x^2+x+1}{x^2(x+1)}$

PRACTICE TEST 4A

1 $\dfrac{a^2}{b+c} = \dfrac{3^2}{-2+5}$

$= \dfrac{9}{3}$

$= 3$ $\therefore$ the answer is **C**.

2 $\dfrac{6a^2b}{4b^2} = \dfrac{{}^{3}\cancel{6} \times a \times a \times \cancel{b}^{1}}{{}_{2}\cancel{4} \times b \times \cancel{b}_{1}}$

$= \dfrac{3a^2}{2b}$ $\therefore$ the answer is **B**.

3 **A** $\dfrac{r-q^2}{p}$

$= \dfrac{-1-(-3)^2}{2}$

$= \dfrac{-1-9}{2}$

$= \frac{-10}{2}$

$= -5$ ✗

B $\dfrac{3p+r}{p+q}$

$= \dfrac{3(2)+(-1)}{2+(-3)}$

$= \dfrac{6-1}{2-3}$

$= \frac{5}{-1}$

$= -5$ ✗

C $\dfrac{p^2-r}{q}$

$= \dfrac{2^2-(-1)}{-3}$

$= \dfrac{4+1}{-3}$

$= \frac{5}{-3}$

$= -\frac{5}{3}$ ✗

D $\dfrac{2q^2+p}{q+r}$

$= \dfrac{2(-3)^2+2}{(-3)+(-1)}$

$= \dfrac{2\times 9+2}{-3-1}$

$= \frac{20}{-4}$

$= -5$ ✗

E $\dfrac{p-q}{r^2}$

$= \dfrac{2-(-3)}{(-1)^2}$

$= \dfrac{2+3}{1}$

$= \frac{5}{1}$

$= 5$ ✓

$\therefore$ the answer is **E**.

4 $\frac{k}{5} \div \frac{3}{k} = \frac{k}{5} \times \frac{k}{3}$

$= \frac{k \times k}{5 \times 3}$

$= \frac{k^2}{15}$ $\therefore$ the answer is **C**.

5 $\frac{4(x+2)(x-3)}{12(x-3)}$

$= \frac{{}^{1}\cancel{4}(x+2)\cancel{(x-3)}^{1}}{{}_{3}\cancel{12}\cancel{(x-3)}_{1}}$

$= \frac{x+2}{3}$ $\therefore$ the answer is **A**.

6 **A** $\frac{x^2+5x+6}{x^2+8x+15}$

$= \frac{(x+2)\cancel{(x+3)}^{1}}{{}_{1}\cancel{(x+3)}(x+5)}$

$= \frac{x+2}{x+5}$ ✗

B $\frac{x^2-2x}{x^2-5x}$

$= \frac{{}^{1}\cancel{x}(x-2)}{{}_{1}\cancel{x}(x-5)}$

$= \frac{x-2}{x-5}$ ✗

C $\frac{x^2-3x+2}{x^2+4x-5}$

$= \frac{{}^{1}\cancel{(x-1)}(x-2)}{{}_{1}\cancel{(x-1)}(x+5)}$

$= \frac{x-2}{x+5}$ ✗

D $\frac{x^2-4}{x^2-7x+10}$

$= \frac{(x+2)\cancel{(x-2)}^{1}}{{}_{1}\cancel{(x-2)}(x-5)}$

$= \frac{x+2}{x-5}$ ✓

$\therefore$ the answer is **D**.

7 $\frac{a}{5} + \frac{3a}{7} = \frac{a \times \mathbf{7}}{5 \times \mathbf{7}} + \frac{3a \times \mathbf{5}}{7 \times \mathbf{5}}$ $\{\text{LCD} = 35\}$

$= \frac{7a}{35} + \frac{15a}{35}$

$= \frac{22a}{35}$ $\therefore$ the answer is **B**.

8 $\frac{m^2-2m}{m+5} \times \frac{5m+25}{3m-6}$

$= \frac{m(m-2)}{m+5} \times \frac{5(m+5)}{3(m-2)}$

$= \frac{m \times {}^{1}\cancel{(m-2)} \times 5 \times \cancel{(m+5)}^{1}}{{}_{1}\cancel{(m+5)} \times 3 \times \cancel{(m-2)}_{1}}$

$= \frac{5m}{3}$ $\therefore$ the answer is **A**.

9 $a - \frac{10}{a} = \frac{a \times \boldsymbol{a}}{1 \times \boldsymbol{a}} - \frac{10}{a}$ $\{\text{LCD} = a\}$

$= \frac{a^2}{a} - \frac{10}{a}$

$= \frac{a^2 - 10}{a}$ $\therefore$ the answer is **E**.

10 $\frac{5}{x+2} - \frac{3}{x-1} = \left(\frac{5}{x+2}\right)\left(\frac{\boldsymbol{x-1}}{\boldsymbol{x-1}}\right) - \left(\frac{3}{x-1}\right)\left(\frac{\boldsymbol{x+2}}{\boldsymbol{x+2}}\right)$ $\{\text{LCD} = (x+2)(x-1)\}$

$= \frac{5(x-1)}{(x+2)(x-1)} - \frac{3(x+2)}{(x+2)(x-1)}$

$= \frac{5(x-1) - 3(x+2)}{(x+2)(x-1)}$

$= \frac{5x - 5 - 3x - 6}{(x+2)(x-1)}$

$= \frac{2x - 11}{(x+2)(x-1)}$ $\therefore$ the answer is **D**.

PRACTICE TEST 4B

1 **a** $\frac{p}{m+n}$

$= \frac{6}{-4+3}$

$= \frac{6}{-1}$

$= -6$

b $\frac{p-2n}{m+n}$

$= \frac{6-2(3)}{-4+3}$

$= \frac{6-6}{-1}$

$= \frac{0}{-1}$

$= 0$

c $\frac{p-m}{\sqrt{m^2+n^2}}$

$= \frac{6-(-4)}{\sqrt{(-4)^2+3^2}}$

$= \frac{6+4}{\sqrt{16+9}}$

$= \frac{10}{\sqrt{25}}$

$= \frac{10}{5}$

$= 2$

2 **a** $\dfrac{(3x)^2}{6x^3}$

$= \dfrac{{}^{1}\cancel{3} \times x^{1}\times 3 \times x^{1}}{{}_{2}\cancel{6} \times x_{1}\times x \times x_{1}}$

$= \dfrac{3}{2x}$

b $\dfrac{3a + 6b}{3}$

$= \dfrac{{}^{1}\cancel{3}(a + 2b)}{\cancel{3}_{1}}$

$= a + 2b$

c $\dfrac{(x + 2)^2}{x^2 + 2x}$

$= \dfrac{{}^{1}\cancel{(x + 2)}(x + 2)}{x\cancel{(x + 2)}_{1}}$

$= \dfrac{x + 2}{x}$

3 **a** $\dfrac{a + b}{3b + 3a}$

$= \dfrac{\cancel{(a + b)}^{1}}{3\cancel{(a + b)}_{1}}$

$= \dfrac{1}{3}$

b $\dfrac{2x^2 - 8}{x + 2}$

$= \dfrac{2(x^2 - 4)}{x + 2}$

$= \dfrac{2\cancel{(x + 2)}^{1}(x - 2)}{\cancel{(x + 2)}_{1}}$

$= 2(x - 2)$

c $\dfrac{x^2 - 6x + 9}{4x - 12}$

$= \dfrac{(x - 3)\cancel{(x - 3)}^{1}}{4\cancel{(x - 3)}_{1}}$

$= \dfrac{x - 3}{4}$

4 **a** $\dfrac{m}{n} \times \dfrac{2}{n} = \dfrac{m \times 2}{n \times n}$

$= \dfrac{2m}{n^2}$

b $\dfrac{m}{n} \div \dfrac{2}{n} = \dfrac{m}{n} \times \dfrac{n}{2}$

$= \dfrac{m \times \cancel{n}^{1}}{{}_{1}\cancel{n} \times 2}$

$= \dfrac{m}{2}$

5 **a** $\dfrac{3}{x} + \dfrac{5}{2x} = \dfrac{3 \times \mathbf{2}}{x \times \mathbf{2}} + \dfrac{5}{2x}$ $\{\text{LCD} = 2x\}$

$= \dfrac{6}{2x} + \dfrac{5}{2x}$

$= \dfrac{11}{2x}$

b $\dfrac{6}{y} - \dfrac{a}{b} = \dfrac{6 \times \boldsymbol{b}}{y \times \boldsymbol{b}} - \dfrac{a \times \boldsymbol{y}}{b \times \boldsymbol{y}}$ $\{\text{LCD} = by\}$

$= \dfrac{6b}{by} - \dfrac{ay}{by}$

$= \dfrac{6b - ay}{by}$

6 **a** $\dfrac{3x}{7} - \dfrac{x}{14} = \dfrac{3x \times \mathbf{2}}{7 \times \mathbf{2}} - \dfrac{x}{14}$ $\{\text{LCD} = 14\}$

$= \dfrac{6x}{14} - \dfrac{x}{14}$

$= \dfrac{5x}{14}$

b $\dfrac{4}{3x} + \dfrac{3}{x^2}$

$= \dfrac{4 \times \boldsymbol{x}}{3x \times \boldsymbol{x}} + \dfrac{3 \times \mathbf{3}}{x^2 \times \mathbf{3}}$ $\{\text{LCD} = 3x^2\}$

$= \dfrac{4x}{3x^2} + \dfrac{9}{3x^2}$

$= \dfrac{4x + 9}{3x^2}$

7 **a** $5 + \dfrac{x}{2} = \dfrac{5 \times \mathbf{2}}{1 \times \mathbf{2}} + \dfrac{x}{2}$ $\{\text{LCD} = 2\}$

$= \dfrac{10}{2} + \dfrac{x}{2}$

$= \dfrac{10 + x}{2}$

b $3 - \dfrac{y}{x} = \dfrac{3 \times \boldsymbol{x}}{1 \times \boldsymbol{x}} - \dfrac{y}{x}$ $\{\text{LCD} = x\}$

$= \dfrac{3x}{x} - \dfrac{y}{x}$

$= \dfrac{3x - y}{x}$

c $1 + \dfrac{x}{2} + \dfrac{y}{3} = \dfrac{1 \times \mathbf{6}}{1 \times \mathbf{6}} + \dfrac{x \times \mathbf{3}}{2 \times \mathbf{3}} + \dfrac{y \times \mathbf{2}}{3 \times \mathbf{2}}$ $\{\text{LCD} = 6\}$

$= \dfrac{6}{6} + \dfrac{3x}{6} + \dfrac{2y}{6}$

$= \dfrac{6 + 3x + 2y}{6}$

8 **a** $\dfrac{y^2-5y}{y+2}\times\dfrac{3}{2y-10}=\dfrac{y(y-5)}{y+2}\times\dfrac{3}{2(y-5)}$

$=\dfrac{y\times{}^{1}\cancel{(y-5)}\times 3}{(y+2)\times 2\times\cancel{(y-5)}_{1}}$

$=\dfrac{3y}{2(y+2)}$

b $\dfrac{9-3x}{4}\div\dfrac{x^2-3x}{6}=\dfrac{9-3x}{4}\times\dfrac{6}{x^2-3x}$

$=\dfrac{3(3-x)\times 6}{4x(x-3)}$

$=\dfrac{-3\cancel{(x-3)}^{1}\times 6^{3}}{{}_{2}\cancel{4x(x-3)}_{1}}$

$=\dfrac{-9}{2x}$

9 **a** $\dfrac{x}{4}-\dfrac{2-x}{8}$

$=\dfrac{x\times\mathbf{2}}{4\times\mathbf{2}}-\dfrac{2-x}{8}\qquad\{\text{LCD}=8\}$

$=\dfrac{2x}{8}-\dfrac{2-x}{8}$

$=\dfrac{2x-(2-x)}{8}$

$=\dfrac{2x-2+x}{8}$

$=\dfrac{3x-2}{8}$

b $\dfrac{x+5}{2}+\dfrac{2x+1}{5}$

$=\left(\dfrac{x+5}{2}\right)\dfrac{\mathbf{5}}{\mathbf{5}}+\left(\dfrac{2x+1}{5}\right)\dfrac{\mathbf{2}}{\mathbf{2}}\qquad\{\text{LCD}=10\}$

$=\dfrac{5(x+5)}{10}+\dfrac{2(2x+1)}{10}$

$=\dfrac{5(x+5)+2(2x+1)}{10}$

$=\dfrac{5x+25+4x+2}{10}$

$=\dfrac{9x+27}{10}$ or $\dfrac{9(x+3)}{10}$

10 **a** $\dfrac{2}{x-1}-\dfrac{3}{x+2}=\left(\dfrac{2}{x-1}\right)\left(\dfrac{\boldsymbol{x+2}}{\boldsymbol{x+2}}\right)-\left(\dfrac{3}{x+2}\right)\left(\dfrac{\boldsymbol{x-1}}{\boldsymbol{x-1}}\right)\qquad\{\text{LCD}=(x-1)(x+2)\}$

$=\dfrac{2(x+2)-3(x-1)}{(x-1)(x+2)}$

$=\dfrac{2x+4-3x+3}{(x-1)(x+2)}$

$=\dfrac{7-x}{(x-1)(x+2)}$

b $\dfrac{1}{x-1}-\dfrac{2}{x^2}=\left(\dfrac{1}{x-1}\right)\dfrac{\boldsymbol{x^2}}{\boldsymbol{x^2}}-\left(\dfrac{2}{x^2}\right)\left(\dfrac{\boldsymbol{x-1}}{\boldsymbol{x-1}}\right)\qquad\{\text{LCD}=x^2(x-1)\}$

$=\dfrac{x^2-2(x-1)}{x^2(x-1)}$

$=\dfrac{x^2-2x+2}{x^2(x-1)}$

PRACTICE TEST 4C

1 **a** $\dfrac{x}{5}+\dfrac{x+2}{4}=\dfrac{6}{5}+\dfrac{6+2}{4}$

$=\frac{6}{5}+\frac{8}{4}$

$=\frac{6}{5}+\frac{2}{1}$

$=\frac{6}{5}+\frac{10}{5}$

$=\frac{16}{5}\quad\left(=3\frac{1}{5}\right)$

b $\dfrac{x}{5}+\dfrac{x+2}{4}$

$=\left(\dfrac{x}{5}\right)\left(\dfrac{\mathbf{4}}{\mathbf{4}}\right)+\left(\dfrac{x+2}{5}\right)\dfrac{\mathbf{5}}{\mathbf{5}}\qquad\{\text{LCD}=20\}$

$=\dfrac{4x}{20}+\dfrac{5(x+2)}{20}$

$=\dfrac{4x+5(x+2)}{20}$

$=\dfrac{4x+5x+10}{20}$

$=\dfrac{9x+10}{20}$

c $10\left(\frac{x}{5}+\frac{x+2}{4}\right)+\frac{x}{2}=10\left(\frac{9x+10}{20}\right)+\frac{x}{2}$ {using **b**}

$$=\frac{10}{1}\times\frac{9x+10}{20}+\frac{x}{2}$$
$$=\frac{{}^{1}\cancel{10}(9x+10)}{{}_{2}\cancel{20}}+\frac{x}{2}$$
$$=\frac{9x+10}{2}+\frac{x}{2}$$
$$=\frac{9x+10+x}{2}$$
$$=\frac{{}^{1}10x+10}{1\,2}$$
$$=\frac{{}^{5}\cancel{10}(x+1)}{{}_{1}\cancel{2}}$$
$$=5(x+1)$$
$$=5x+5$$

2 **a** If Ivan scored x goals for the season, then $\frac{x}{2}$ of his goals came from penalties.

Rico scored $(x+8)$ goals, so $\frac{x+8}{5}$ of his goals came from penalties.

So, the total number of penalty goals scored $=\frac{x}{2}+\frac{x+8}{5}$

$$=\left(\frac{x}{2}\right)\left(\frac{\mathbf{5}}{\mathbf{5}}\right)+\left(\frac{x+8}{5}\right)\left(\frac{\mathbf{2}}{\mathbf{2}}\right) \quad \{\text{LCD}=10\}$$
$$=\frac{5x+2(x+8)}{10}$$
$$=\frac{5x+2x+16}{10}$$
$$=\frac{7x+16}{10}$$

b **i** If $x=12$, $\frac{7x+16}{10}=\frac{7(12)+16}{10}$

$$=\frac{84+16}{10}$$
$$=\frac{100}{10}$$
$$=10$$

So, Ivan and Rico scored 10 penalty goals between them.

ii If $x=22$, $\frac{7x+16}{10}=\frac{7(22)+16}{10}$

$$=\frac{154+16}{10}$$
$$=\frac{170}{10}$$
$$=17$$

So, Ivan and Rico scored 17 penalty goals between them.

3 **a** **i** $a-\frac{9}{a}=\frac{a\times\boldsymbol{a}}{1\times\boldsymbol{a}}-\frac{9}{a}$ $\{\text{LCD}=a\}$

$$=\frac{a^2-9}{a}$$

ii $1-\frac{a}{3}=\frac{1\times\mathbf{3}}{1\times\mathbf{3}}-\frac{a}{3}$ $\{\text{LCD}=3\}$

$$=\frac{3-a}{3}$$

b $\left(a-\frac{9}{a}\right)\div\left(1-\frac{a}{3}\right)=\frac{a^2-9}{a}\div\frac{3-a}{3}$ {using **a**}

$=\frac{a^2-9}{a}\times\frac{3}{3-a}$

$=\frac{(a+3)(a-3)}{a}\times\frac{3}{-1(a-3)}$

$=\frac{(a+3)\times\cancel{(a-3)}^{1}\times 3}{a\times -1\times\cancel{(a-3)}_{1}}$

$=\frac{3(a+3)}{-a}$

$=-\frac{3(a+3)}{a}$

c Using **b**, $\left(a-\frac{9}{a}\right)\div\left(1-\frac{a}{3}\right)=\frac{3(a+3)}{-a}$

i When $a=1$,

$\frac{3(a+3)}{-a}=\frac{3(1+3)}{-1}$

$=\frac{3\times 4}{-1}$

$=\frac{12}{-1}$

$=-12$

ii When $a=3$,

$\frac{3(a+3)}{-a}=\frac{3(3+3)}{-3}$

$=\frac{{}^{1}\cancel{3}\times 6}{-1\cancel{-3}}$

$=\frac{6}{-1}$

$=-6$

iii When $a=5$,

$\frac{3(a+3)}{-a}=\frac{3(5+3)}{-5}$

$=\frac{3\times 8}{-5}$

$=\frac{24}{-5}$

$=-\frac{24}{5}$

4 **a** **i** $(x+y)\div\left(\frac{1}{x}+\frac{1}{y}\right)$

$=(3+4)\div\left(\frac{1}{3}+\frac{1}{4}\right)$

$=7\div\left(\frac{4}{12}+\frac{3}{12}\right)$

$=\frac{7}{1}\div\frac{7}{12}$

$=\frac{{}^{1}\cancel{7}}{1}\times\frac{12}{\cancel{7}_{1}}$

$=12$

ii $(x+y)\div\left(\frac{1}{x}+\frac{1}{y}\right)$

$=(5+10)\div\left(\frac{1}{5}+\frac{1}{10}\right)$

$=15\div\left(\frac{2}{10}+\frac{1}{10}\right)$

$=\frac{15}{1}\div\frac{3}{10}$

$=\frac{{}^{5}\cancel{15}}{1}\times\frac{10}{\cancel{3}_{1}}$

$=50$

b It appears that $(x+y)\div\left(\frac{1}{x}+\frac{1}{y}\right)$ simplifies to $x\times y=xy$.

c $\frac{1}{x}+\frac{1}{y}=\frac{1\times\boldsymbol{y}}{x\times\boldsymbol{y}}+\frac{1\times\boldsymbol{x}}{y\times\boldsymbol{x}}$ {LCD $=xy$}

$=\frac{y}{xy}+\frac{x}{xy}$

$=\frac{x+y}{xy}$

So, $(x+y)\div\left(\frac{1}{x}+\frac{1}{y}\right)=\frac{x+y}{1}\div\frac{x+y}{xy}$

$=\frac{{}^{1}\cancel{(x+y)}}{1}\times\frac{xy}{{}_{1}\cancel{(x+y)}}$

$=xy$

This is what we predicted in **b**.

d $\frac{41}{\frac{1}{21}+\frac{1}{20}}=41\div\left(\frac{1}{21}+\frac{1}{20}\right)$

$=(21+20)\div\left(\frac{1}{21}+\frac{1}{20}\right)$

which is in the form $(a+b)\div\left(\frac{1}{a}+\frac{1}{b}\right)$ where $a=21$ and $b=20$

$\therefore\ \frac{41}{\frac{1}{21}+\frac{1}{20}}=21\times 20$ {using **c**}

$=420$

5 **a** **i** $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}$

$=\frac{3}{4}$

And $\frac{2}{3}-\frac{3}{4}=\frac{8}{12}-\frac{9}{12}$

$=-\frac{1}{12}$

So, $\frac{3}{4}>\frac{2}{3}$

$\therefore\ \frac{1}{2}+\frac{1}{4}>\frac{1}{3}+\frac{1}{3}.$

ii $\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$

$\frac{1}{4}+\frac{1}{6}=\frac{3}{12}+\frac{2}{12}$

$=\frac{5}{12}$

And $\frac{2}{5}-\frac{5}{12}=\frac{24}{60}-\frac{25}{60}$

$=-\frac{1}{60}$

So, $\frac{5}{12}>\frac{2}{5}$

$\therefore\ \frac{1}{4}+\frac{1}{6}>\frac{1}{5}+\frac{1}{5}.$

iii $\frac{1}{9}+\frac{1}{9}=\frac{2}{9}$

$\frac{1}{8}+\frac{1}{10}=\frac{5}{40}+\frac{4}{40}$

$=\frac{9}{40}$

And $\frac{2}{9}-\frac{9}{40}=\frac{80}{360}-\frac{81}{360}$

$=-\frac{1}{360}$

So, $\frac{9}{40}>\frac{2}{9}$

$\therefore\ \frac{1}{8}+\frac{1}{10}>\frac{1}{9}+\frac{1}{9}.$

b For any positive integer $x>1$, it appears that $\frac{1}{x-1}+\frac{1}{x+1}$ is greater than $\frac{1}{x}+\frac{1}{x}$.

c Now, $\frac{1}{x}+\frac{1}{x}=\frac{2}{x}$, so the difference between $\frac{1}{x-1}+\frac{1}{x+1}$ and $\frac{2}{x}$ is

$$\frac{1}{x-1}+\frac{1}{x+1}-\frac{2}{x}=\left(\frac{1}{x-1}\right)\left(\frac{\boldsymbol{x(x+1)}}{\boldsymbol{x(x+1)}}\right)+\left(\frac{1}{x+1}\right)\left(\frac{\boldsymbol{x(x-1)}}{\boldsymbol{x(x-1)}}\right)-\frac{2}{x}\left(\frac{\boldsymbol{(x-1)(x+1)}}{\boldsymbol{(x-1)(x+1)}}\right)$$

$\{\text{LCD}=x(x-1)(x+1)\}$

$$=\frac{x(x+1)+x(x-1)-2(x-1)(x+1)}{x(x-1)(x+1)}$$

$$=\frac{x^2+x+x^2-x-2(x^2-1)}{x(x-1)(x+1)}$$

$$=\frac{\cancel{x^2}+\cancel{x}+\cancel{x^2}-\cancel{x}-\cancel{2x^2}+2}{x(x-1)(x+1)}$$

$$=\frac{2}{x(x-1)(x+1)}$$

2 is clearly positive and since $x>1$, x, $x-1$, and $x+1$ are all positive as well.

So, $\frac{2}{x(x-1)(x+1)}$ is positive for all $x>1$.

$\therefore\ \frac{1}{x-1}+\frac{1}{x+1}$ is greater than $\frac{1}{x}+\frac{1}{x}$ for any positive integer $x>1$.